



**Course: Power System Analysis 1– BEE601**

**Module-2: Symmetrical Fault Analysis**

**Maria Sushma S**  
**Assistant Professor**  
**Dept. of Electrical and Electronics Engineering**  
**ATME College of Engineering, Mysuru**

## MODULE- 2 Symmetrical Fault Analysis:

- Types of Faults
- Nature And Causes of Faults, Faults Analysis
- Symmetrical Faults Analysis
- Transient Due To Short Circuit In Transmission Line
- Short Circuit Transient In Synchronous Machine (on No Load)
- Short Circuit Of A Loaded Synchronous Machine
- Symmetrical Fault Analysis
  - ❖ Symmetrical fault current estimation using Kirchoff's laws
  - ❖ Symmetrical fault current estimation using thevenin's theorem
  - ❖ Numerical
- Selection Of Circuit Breakers

## Faults Analysis

A **fault** in a circuit is any failure that interferes with the normal flow of current to the load.

In most faults, a short circuit path forms between two or more phases, or between one or more phases and the neutral (ground). Since the impedance of a new path is usually low, an excessive current may flow.

### Reason for faults in power system

Faults occur in a power system due to insulation failure of equipments, flashover of lines initiated by a lightening stroke, permanent damage to conductors and towers or accidental faulty operations

## Nature and causes of Faults

| Equipment                  | Cause of fault  | % of Total Faults |
|----------------------------|---|-------------------|
| 1. Overhead lines          | Lightning strokes<br>Storms, earthquakes, icing<br>Birds, trees, kites aero planes, snakes, etc.<br>Internal over-voltages.                             | 30—40             |
| 2. Underground cables      | Damage during digging<br>Insulation failure due to temperature rise<br>Failure of joints  | 8—10              |
| 3, Alternators (Generator) | Stator faults<br>Rotor faults<br>Abnormal conditions<br>Faults in associated equipment<br>Faults in protective system                                   | 6—8               |
| 4. Transformers            | Insulation failure<br>Faults in tap-changer<br>Faults in bushing<br>Faults in protection circuit<br>Inadequate protection<br>Overloading, over voltage. | 10—12             |
| 5. CT, PT                  | Over-voltages<br>Insulation failures<br>Breaking of conductors<br>Wrong connections   | 15-20             |
| 6 Switchgear               | Insulation failure<br>Mechanical defect<br>Leakage of air/oil/gas<br>Inadequate rating<br>Lack of maintenance.  | 10-12             |

## Faults analysis

Fault currents cause equipment damage due to both thermal and mechanical processes.

The main goal of fault analysis is to determine the magnitudes of the currents present during the fault:

For proper choice of circuit breakers and protective relaying, we must estimate the magnitude of currents that would flow under short circuit conditions-this is the scope of fault analysis (study)

### **The Reason to analyze faults are:**

- We need to determine the maximum current to ensure devices can survive the fault.
- We need to determine the maximum current the circuit breakers (CBs) need to interrupt to correctly size the CBs.
- To set the relays so that can detect it.
- To make sure that the circuit breakers ratings are such that they are capable of interrupting the fault current

## TYPES OF FAULTS

Two broad classifications of faults are

1. Symmetrical faults
2. Unsymmetrical faults

### Statistics of fault

|                      | Types of fault                                   | % of occurrence |
|----------------------|--|-----------------|
| Unsymmetrical faults | Single Line to ground faults                     | 70%             |
|                      | Double line faults                               | 15%             |
|                      | Double line to ground faults                     | 10%             |
| symmetrical faults   | Triple line faults (Balanced three phase faults) | 5%              |

**Symmetrical faults:** Fault current is same in all the Three Phases and hence system remains balanced even after fault occurrence, symmetrical fault conditions are analyzed on a single phase basis using thevenin's theorem or using bus matrix impedance matrix. these faults are relatively rare, but are the easiest to analyze so we'll consider them first

**Unsymmetrical faults:** Fault current is not same in all the Three Phases , System is no longer balanced; these faults are very common, but more difficult to analyze. Analyzed using symmetrical components

In a power system, the most severe fault is three phase fault and less severe fault is open conductor fault. **The various faults in the order of decreasing severity are,**

- 1) Three phase fault
- 2) Double line-to-ground fault
- 3) Line-to-line fault
- 4) Single line-to-ground fault
- 5) Open conductor fault

## **Fault calculations**

The fault condition of a power system can be divided into subtransient, transient, and steady state periods. The currents in the various parts of the system and in the fault locations are different in these periods. The estimation of these currents for various types of faults at various locations in the system is commonly referred to as fault calculations

## Symmetrical Faults analysis

We now switch over to abnormal system behaviour under conditions of symmetrical short circuit (symmetrical three-phase fault). Such conditions are caused in the system accidentally through insulation failure of equipment or flashover of lines initiated by a lightning stroke or through accidental faulty operation

The system must be protected against flow of heavy short circuit currents (which can cause permanent damage to major equipment) by disconnecting the faulty part of the system by means of circuit breakers operated by protective relaying. For proper choice of circuit breakers and protective relaying, we must estimate the magnitude of currents that would flow under short circuit conditions-this is the scope of fault analysis (study).

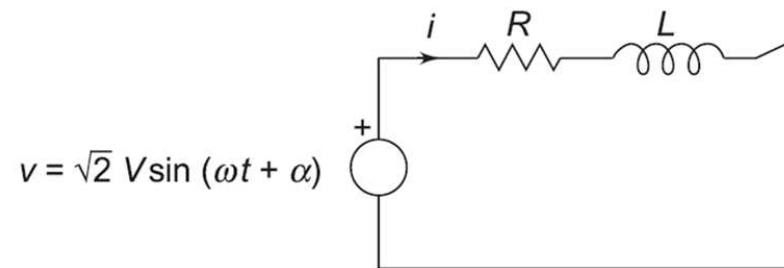


## Transient due To Short Circuit In Transmission Line

Let us consider the short circuit transient on a transmission line. Certain simplifying assumptions are made at this stage, they are as follows

- The line is fed from a constant voltage source.
- Short circuit takes place when the line is unloaded.
- Line capacitance is negligible and the line can be represented by a lumped RL series Circuit.

With the above assumption the line can be represented by the circuit Model as shown below



The short circuit is assumed to take place at  $t = 0$ . The parameter  $\alpha$  controls the instant on the voltage wave when short circuit occurs. It is known from circuit theory that the current after short circuit is composed of two parts, i.e.

$$i = i_s + i_t \quad \text{Where } i_s = \text{steady state current}$$

$$i = i_s + i_t$$

Where  $i_s$  = steady state current

$$= \frac{\sqrt{2}V}{|Z|} \sin (\omega t + \alpha - \theta)$$

$$Z = (R^2 + \omega^2 L^2)^{1/2} \angle \left( \theta = \tan^{-1} \frac{\omega L}{R} \right)$$

$i_t$  = transient current [it is such that  $i(0) = i_s(0) + i_t(0) = 0$  being an inductive circuit; it decays corresponding to the time constant  $L/R$ ].

$$= - i_s(0) e^{-(R/L)t}$$

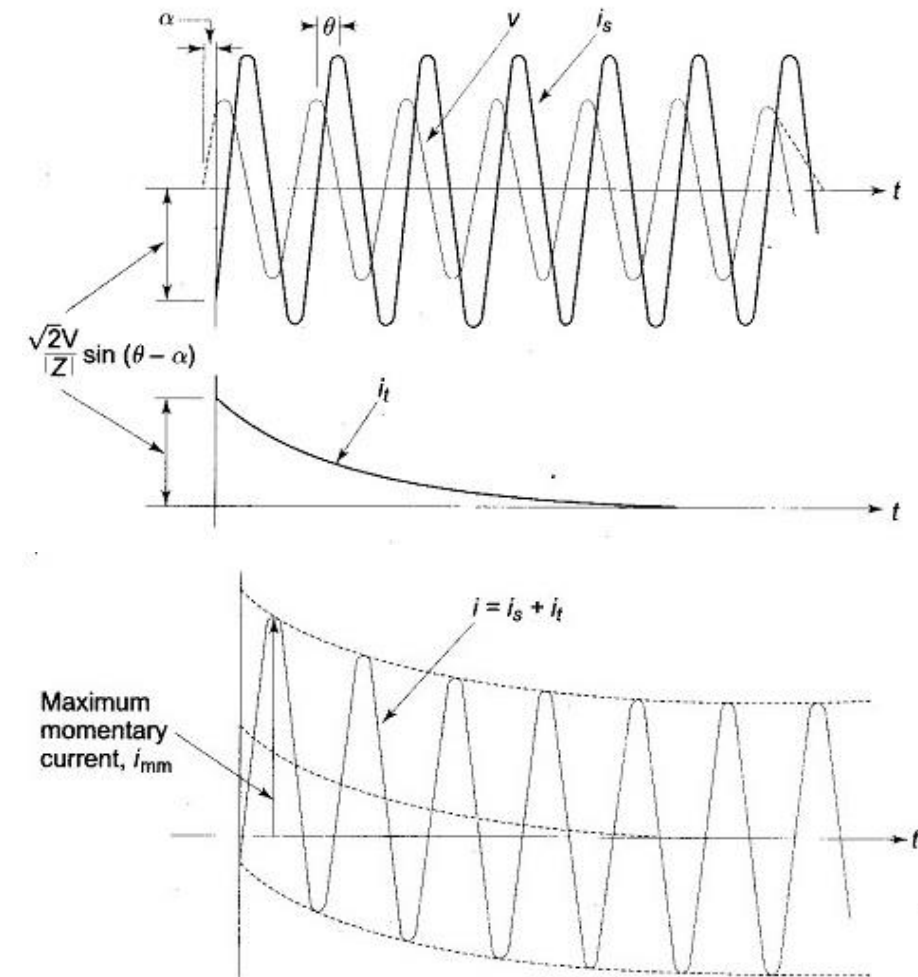
$$= \frac{\sqrt{2}V}{|Z|} \sin (\theta - \alpha) e^{-(R/L)t}$$

Thus short circuit current is given by

$$i = \underbrace{\frac{\sqrt{2}V}{|Z|} \sin (\omega t + \alpha - \theta)}_{\text{Symmetrical short circuit current}} + \underbrace{\frac{\sqrt{2}V}{|Z|} \sin (\theta - \alpha) e^{-(R/L)t}}_{\text{DC off-set current}}$$

A plot of  $i_s$ ,  $i_t$  and  $i = i_s + i_t$  is shown in below Fig. it is observed that the short circuit current has two components, they are sinusoidal steady state current and unidirectional transient component

In power system terminology, the sinusoidal steady state current is called the symmetrical short circuit current and the unidirectional transient component is called the DC off-set current, which causes the total short circuit current to be unsymmetrical till the transient decays.



Waveform of a short circuit current on a transmission line

In the short circuit current  $I$ , the **maximum momentary short circuit current**  $i_{mm}$  corresponds to the first peak. If the decay of transient current in this short time is neglected,

$$i_{mm} = \frac{\sqrt{2}V}{|Z|} \sin(\theta - \alpha) + \frac{\sqrt{2}V}{|Z|}$$

Since transmission line resistance is small,  $\theta \approx 90^\circ$

$$i_{mm} = \frac{\sqrt{2}V}{|Z|} \cos \alpha + \frac{\sqrt{2}V}{|Z|}$$

This has the maximum possible value for  $\alpha = 0$ , i.e. short circuit occurring when the voltage wave is going through zero. Thus

$$i_{mm \text{ (max possible)}} = 2 \frac{\sqrt{2}V}{|Z|}$$

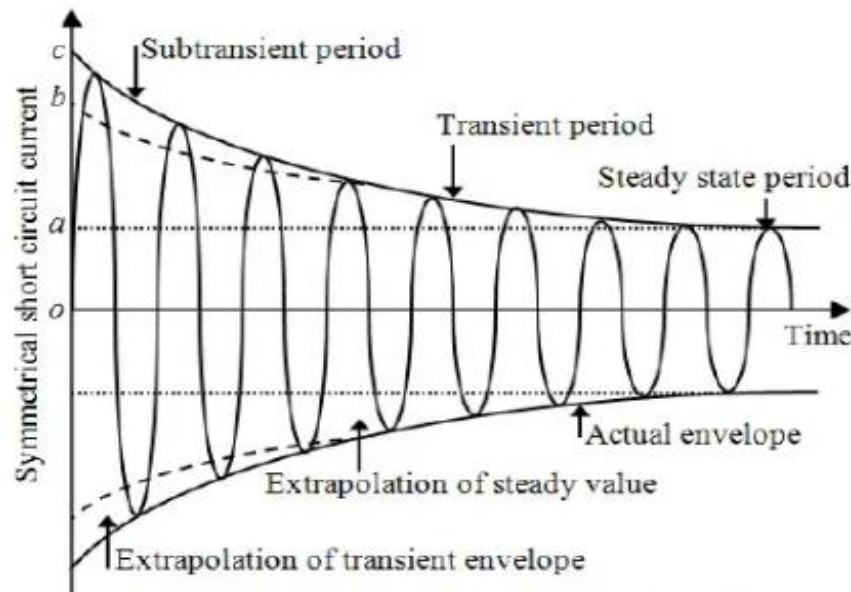
= twice the maximum of symmetrical short circuit current  
*(doubling effect)*

For the selection of circuit breakers, momentary short circuit current is taken corresponding to its maximum possible value( a safe choice).

## Short Circuit Transient in Synchronous Machine (on no load)

- Under steady state SC conditions, the armature reaction of a synchronous generator produces a demagnetizing flux.
- In term of a circuit this effect is modeled as a reactance  $X_a$  in series with the induced emf.
- This reactance when combined with the leakage reactance  $X_l$  of the machine is called synchronous reactance  $X_d$  (direct axis synchronous in the case of salient pole machines). Armature resistance being small can be neglected.

### Oscillogram of short circuit current of a synchronous generator, operating on no load



(a) Symmetrical short-circuit armature current in synchronous machine

## Short Circuit Transient in Synchronous Machine (on no load)

Oscillogram of short circuit current of a synchronous generator, operating on no load

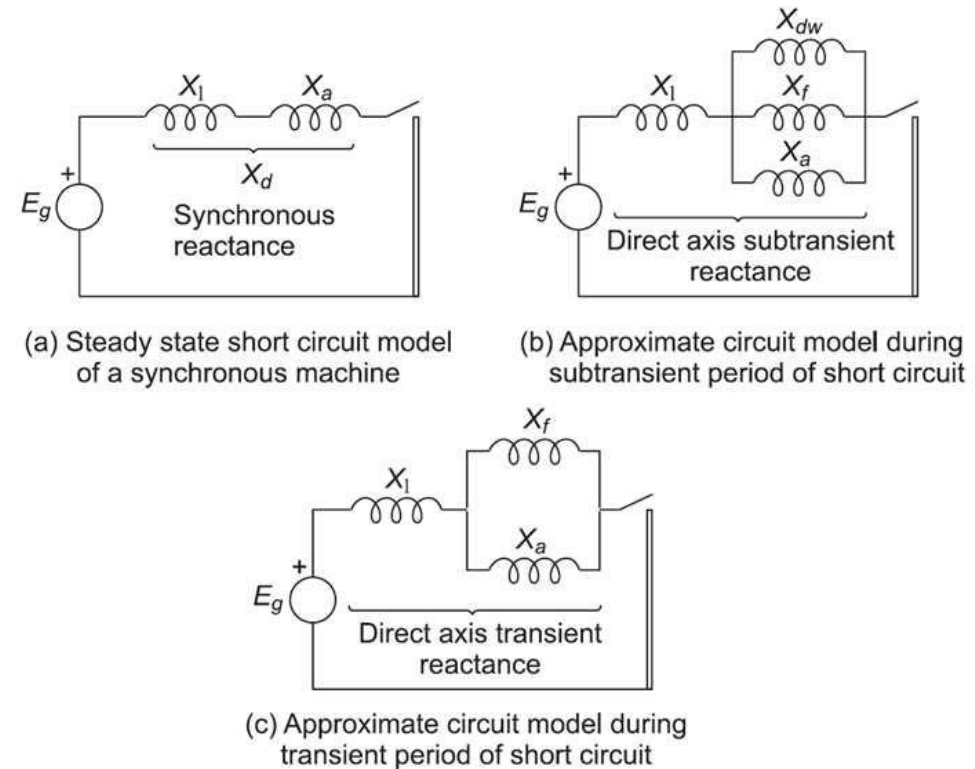
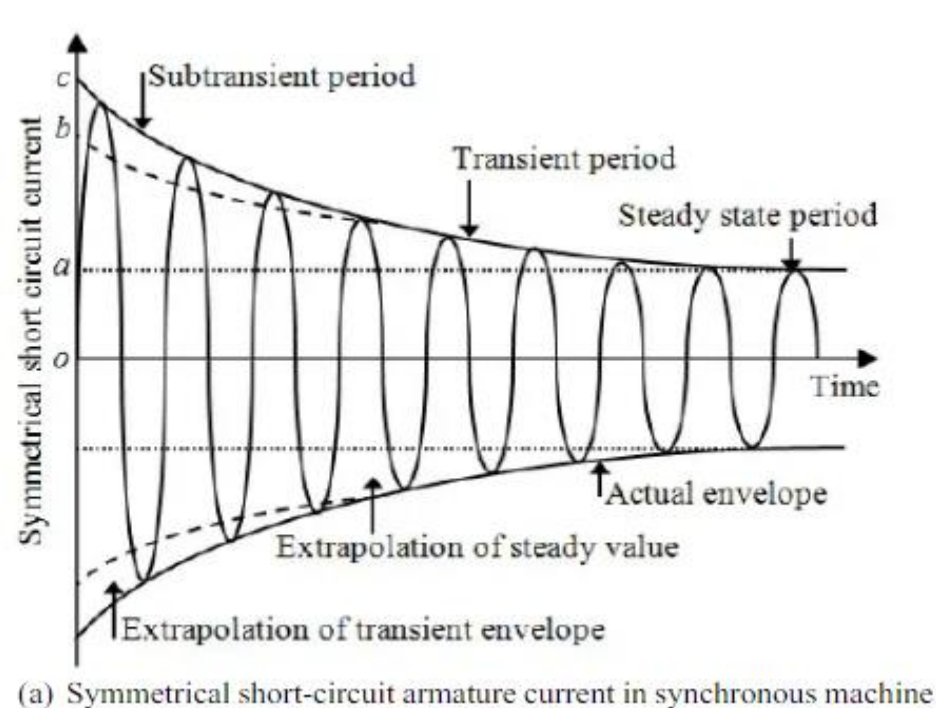


Fig.3

$X_a$  = armature reactance

$X_f$  = field winding reactance

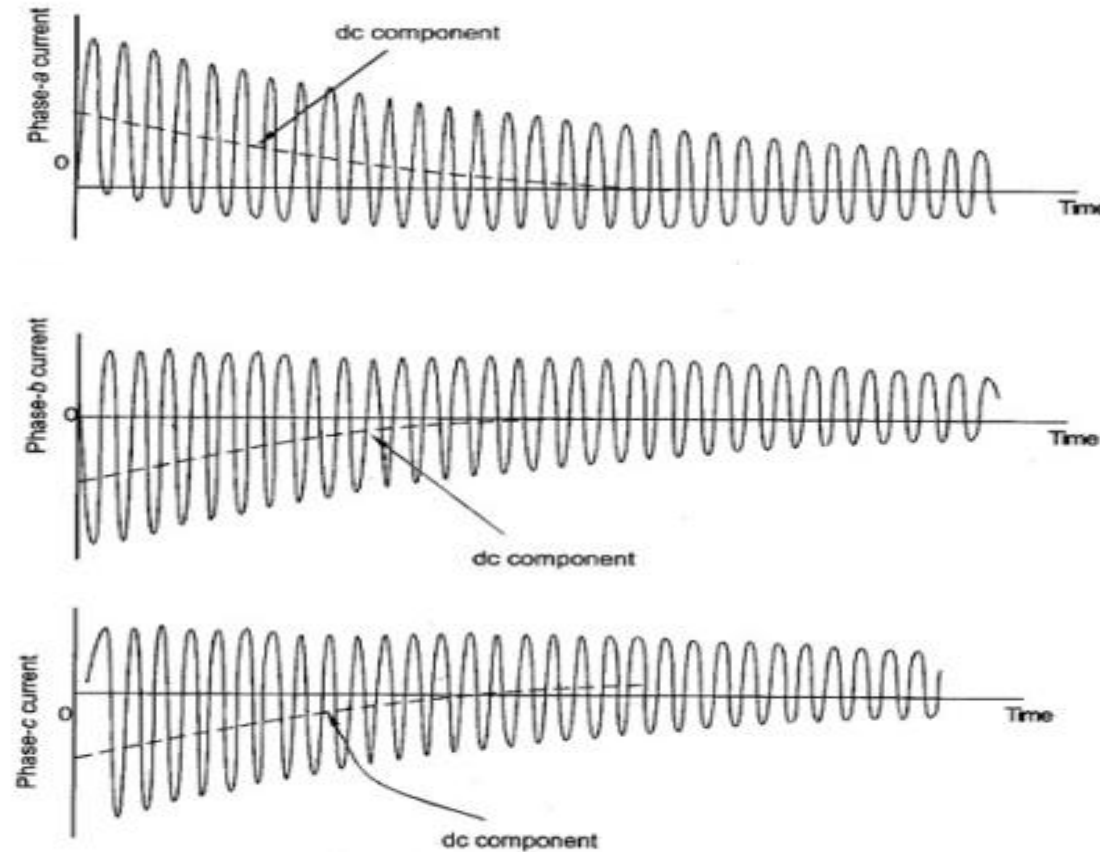
$X_{dw}$  = damper winding reactance

$X_l$  = leakage flux reactance



- Consider now the sudden SC (3-phase) of a synchronous generator initially operating under open circuit conditions. The machine undergoes a transient in all the three-phase finally ending up in a steady state conditions.
- The circuit breaker must interrupt the current much before steady conditions are reached. Immediately upon SC, the DC off-set currents appear in all three-phases, each with a different magnitude since the point on the voltage wave at which SC occurs is different for each phase.
- These DC off-set currents are accounted for separately on an empirical basis and therefore for SC studies, we concentrate the attention on symmetrical (sinusoidal) SC current only.
- Immediately in the event of a SC, the symmetrical SC current is limited only by the leakage reactance of the machine

For short circuit at this instant the wave forms of short circuit currents in the three phases are shown in Fig. which also indicates the dc off-set currents in dotted

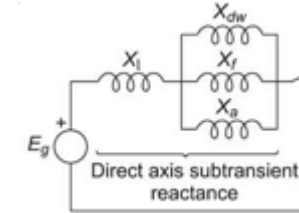


**Fig.** Short circuit current wave forms in the three phases of a synchronous generator



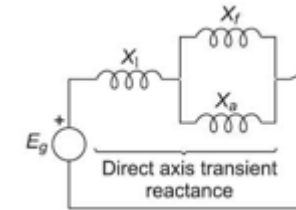
The reactance presented by the machine in the initial period of the SC, i.e.: is called the subtransient reactance of the machine.

$$X_d'' = X_l + \frac{1}{\left(1/X_a + 1/X_f + 1/X_{dw}\right)}$$



While the reactance effective after the damper winding currents have died out, i.e.: is called the transient reactance of the machine.

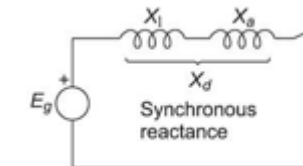
$$X_d' = X_l + (X_a \parallel X_f)$$



The reactance under steady conditions is the synchronous reactance of the machine.

Obviously  $X_d'' < X_d' < X_d$ .

The machine thus offers a time-varying reactance which changes from  $X_d''$  to  $X_d'$  and finally  $X_d$



$X_d$  is the direct axis synchronous reactance

$X_d'$  is the direct axis transient reactance  $X_d'' < X_d' < X_d$

$X_d''$  is the direct axis subtransient reactance

The reactance, which is called **subtransient reactance**, presented by the machine in the initial period of the short-circuit, i.e.

$$X_d'' = X_l + X_a // X_f // X_{dw} = X_l + \frac{1}{[(1/X_a) + (1/X_f) + (1/X_{dw})]}$$

The reactance, which is called **transient reactance**, after the damper winding currents have died out is:

$$X_d' = X_l + X_a // X_f = X_l + \frac{1}{[(1/X_a) + (1/X_f)]}$$

The reactance, which is called **synchronous reactance**, after the field winding currents have died out is:

$$X_d = X_l + X_a$$

Obviously,  $X_d'' < X_d' < X_d$ . The machine thus offers a time-varying reactance which changes from  $X_d''$  to  $X_d'$  and finally to  $X_d$ .

The rms value of *steady-state current*:  $|I| = \frac{oa}{\sqrt{2}} = \frac{E_g}{X_d}$

The rms value of *transient current* excluding dc off-set component:  $|I| = \frac{ob}{\sqrt{2}} = \frac{E_g}{X'_d}$

The rms value of *subtransiente current* excluding dc off-set componnet:  $|I''| = \frac{oc}{\sqrt{2}} = \frac{E_g}{X''_d}$

$X_d$  = direct-axis synchronous reactance

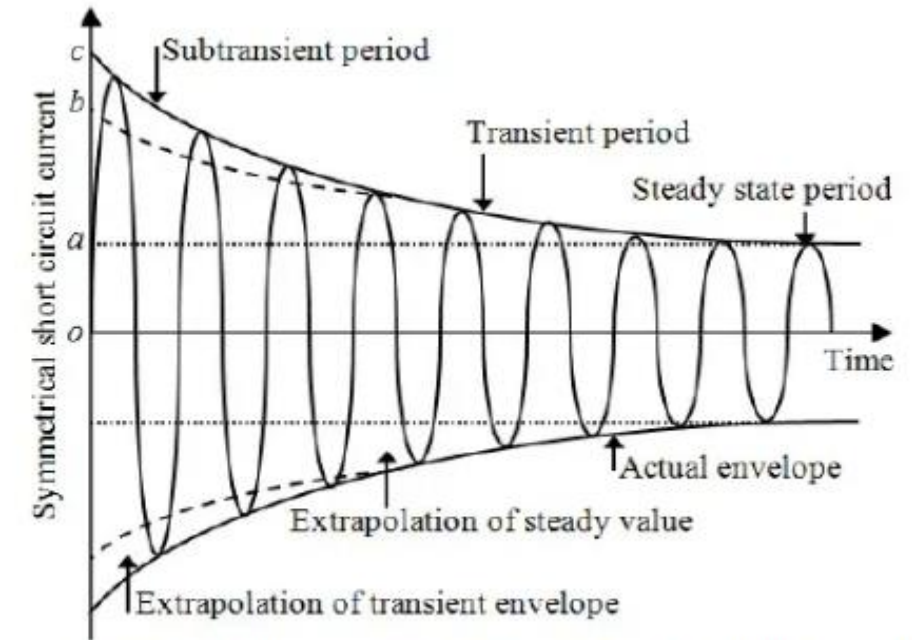
$X'_d$  = direct-axis transient reactance

$X''_d$  = direct-axis subtransient reactance

$|E_g|$  = rms voltage from one terminal to neutral at no load

$oa$ ,  $ob$ , and  $oc$  = intercepts shown in Fig.

Obviously,  $|I''| > |I'| > |I|$

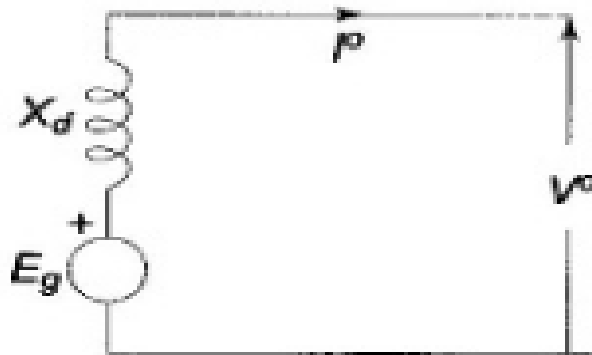


(a) Symmetrical short-circuit armature current in synchronous machine

### Short Circuit of a Loaded Synchronous Machine:

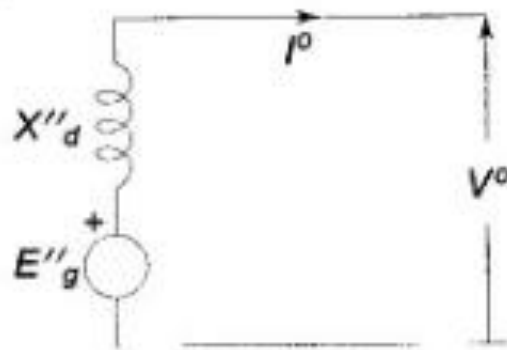
Computing short circuit current when short circuit occurs under loaded conditions

Below fig. shows the circuit model of a synchronous generator operating under steady conditions supplying a load current  $I^\circ$  to the bus at a terminal voltage of  $V^\circ$ .  $E_g$  is the induced emf under loaded condition and  $X_d$  is the direct axis synchronous reactance of the machine.

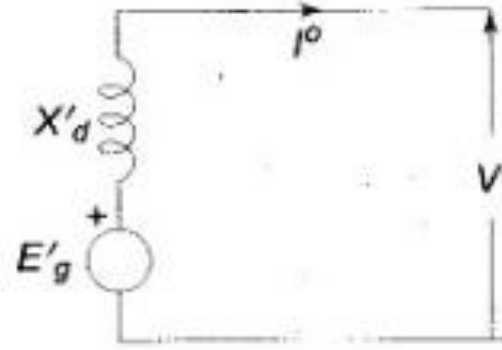


**Fig.** Circuit model of  
a loaded  
machine

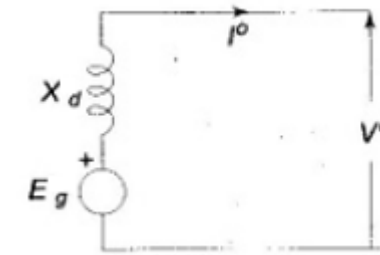
When short circuit occurs at the terminals of this machine, the circuit model to be used for computing short circuit current is given in Fig. a for subtransient current, and in Fig. b for transient current.



(a) Circuit model for computing subtransient current



(b) Circuit model for computing transient current



Circuit Model for Computing steady state current

The induced emfs to be used in these models are given

$$E''_g = V''_0 + jI''_0 X''_d$$

$$E'_g = V'_0 + jI'_0 X'_d$$

The voltage  $E''_g$  is known as the *voltage behind the subtransient reactance* and the voltage  $E'_g$  is known as the *voltage behind the transient reactance*.

In fact, if  $I''_0$  is zero (no load case),  $E''_g = E'_g = E_g$ , the no load voltage, in which case the circuit model reduces

Synchronous motors have internal emfs and reactances similar to that of a generator except that the current direction is reversed. During short circuit conditions these can be replaced by similar circuit models except that the voltage behind subtransient/transient reactance is given by

$$E''_m = V^o - jI^o X''_d$$
$$E'_m = V^o - jI^o X'_d$$

Whenever we are dealing with short circuit of an interconnected system, the synchronous machines (generators and motors) are replaced by their corresponding circuit models having voltage behind subtransient (transient) reactance in series with subtransient (transient) reactance. The rest of the network being passive remains unchanged.

## Symmetrical Fault Analysis

### Method 1

### Symmetrical fault current estimation using Kirchoff's laws

The following procedure can be followed to directly calculate the voltages and currents during symmetrical fault condition in a power system, using kirchoff's laws.

1. Choose appropriate base values and determine the prefault condition reactance diagram of the given power system. [the prefault condition(**just before the fault**) reactance diagram is separately formed for sub transient, transient and steady state condition of the fault]
2. Calculate the internal emfs of synchronous machines(Generators & Motors) and the prefault voltage at the fault point 'F' using prefault current (load current). **[ i.e. just before the fault ]**

**Note:** If the power system is unloaded (i.e., if there is no prefault current then prefault voltage at the fault point is 1 p.u. Also the internal emfs for subtransient and transient state are same as steady state induced emf.

3. Draw the fault condition reactance diagram of the system. This diagram is same as prefault reactance diagram except that the fault 'F' is represented by a short circuit or by the specified fault impedance. The currents in this reactance diagram are fault condition currents.
4. Calculate the p.u. value of fault currents in the various parts of the system and in the fault.
5. The actual values of the fault currents are obtained by multiplying the p.u. values by the respective base currents

**Note:** When transformers are used in the power system, the base currents will be different for various sections of power system.



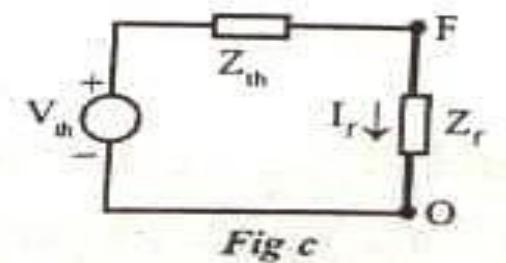
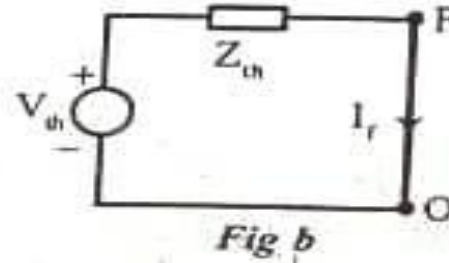
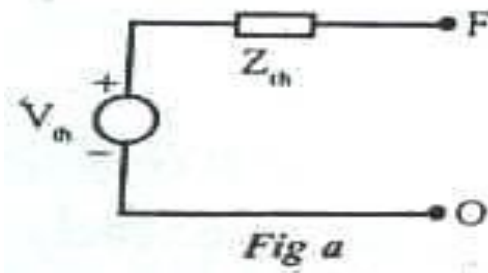
## Method 2

### Symmetrical fault current estimation using thevenin's theorem

The following procedure can be followed to calculate the voltages and currents during symmetrical fault using thevenin's theorem.

1. Choose appropriate base values and determine the prefault condition reactance diagram of the given power
2. Calculate the prefault voltage at the fault point 'F' using the prefault current (load current). If the system is unloaded, then the prefault voltage is 1 p.u. The prefault voltage at the fault point is the thevenin's voltage.
3. Determine the thevenin's impedance of the system at the fault point 'F'. (This is given by looking back impedance at the fault point). To determine the thevenin's impedance, replace all the source by zero value source (i.e by short circuit) and then reduce the resultant network to single equivalent impedance.

4. Draw the thevenin's equivalent at the fault point, F as shown in below fig a. Here the fault 'F' can be represented by a short circuit or by fault impedance as shown in fig b or fig c



Now the current flowing through the short is the fault current

P.U. value of fault current is given by ,  $I_f = V_{th}/Z_{th}$

The actual value of fault current is obtained by multiplying the p.u. value with base current.

5. The fault currents in other parts of the network are determined from the knowledge of change in current due to fault (current delivered by thevenin's generator) and prefault current (Load Current). The fault current (i.e., post fault current) in any part of the system is given by sum of prefault current and change in current due to fault.

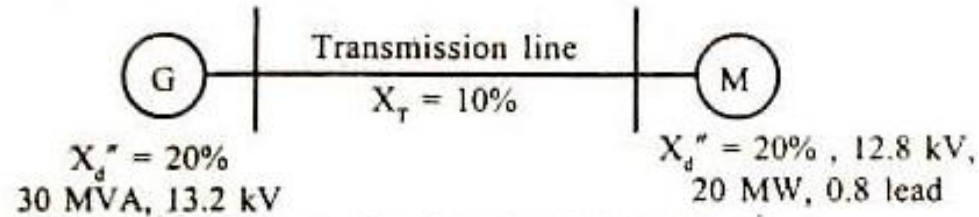
6. The change in current due to fault can be Calculated by connecting the thevenin's source with reversed polarity (i.e. Negative of thevenin's voltage source) at the fault 'F'. Replace all other sources by zero value sources i.e voltage sources are replaced by Short circuit.

Now the currents in various parts of the system are the change in currents due to fault. Calculate these currents by any conventional technique.

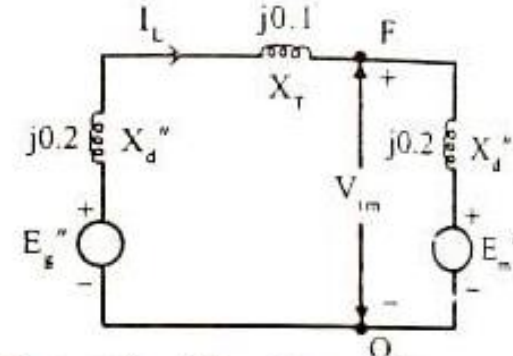
## Numerical 1

A synchronous generator and motor are rated for 30,000 kVA, 13.2 kV and both have subtransient reactance of 20%. The line connecting them has a reactance of 10% on the base of machine ratings. The motor is drawing 20,000 k W at 0.8 pf leading. The terminal voltage of the motor is 12.8 kV. When a symmetrical three-phase fault occurs at motor terminals, find the subtransient current in generator, motor and at the fault point,

The single line diagram and the circuit model to compute subtransient fault current are shown in Fig 1 and 2. respectively.

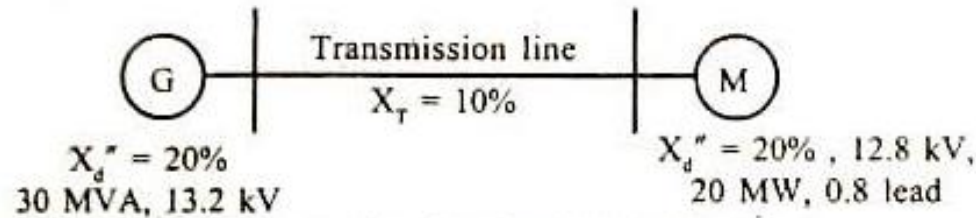


**Fig 1 : Single line diagram**



**Fig 2 : Circuit model to compute subtransient fault current**

In Fig 2. the generator and motor are represented by a source in series with subtransient reactance. The value of the sources are subtransient internal voltages.



**Fig 1 : Single line diagram**

The base values are,  $MVA_b = 30 \text{ MVA}$  ;  $kV_b = 13.2 \text{ kV}$

$$\text{Base current, } I_b = \frac{kVA_b}{\sqrt{3}kV_b} = \frac{MVA_b \times 1000}{\sqrt{3} \times kV_b} = \frac{30 \times 1000}{\sqrt{3} \times 13.2} = 1312.16 \text{ A}$$

Actual value of prefault voltage at fault point,  $V_{tm} = 12.8 \text{ kV}$

$$\text{p.u. value of prefault voltage at fault point, } V_{tm} = \frac{\text{Actual value}}{\text{Base value}} = \frac{12.8}{13.2} = 0.9697 \text{ p.u.}$$

Actual value of real power of the load,  $P_m = 20 \text{ MW}$ , 0.8 lead

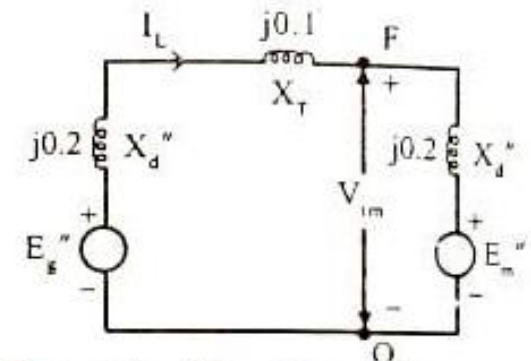
$$\text{p.u. value of real power of the load, } P_m = \frac{\text{Actual value}}{\text{Base value}} = \frac{20}{30} = 0.6667 \text{ p.u.}$$

When voltage, current and power are expressed in p.u., then in 3-phase circuits

$$P = VI \cos \phi$$

where  $\cos \phi$  = power factor of the load

$$\therefore \text{p.u. value of magnitude of load current, } |I| = \frac{P_m}{V_{tm} \cos \phi} = \frac{0.6667}{0.9697 \times 0.8} = 0.8594 \text{ p.u.}$$



**Fig 2 : Circuit model to compute subtransient fault current**

Alternatively the load current can be calculated as shown below

$$\text{Load kVA} = \frac{\text{Read Power}}{\text{pf}} = \frac{20}{0.8} = 25 \text{ kVA}$$

$$\text{p.u value of load kVA, } S_m = \frac{\text{Load kVA}}{\text{Base kVA}} = \frac{25}{30} = 0.8333 \text{ p.u.}$$

$$\text{p.u value of load current, } |I| = \frac{S_m}{V_{tm}} = \frac{0.8333}{0.9697} = 0.8593 \text{ p.u.}$$

Let us take the terminal voltage of motor  $V_{tm}$  as reference vector and so the load current will lead the terminal voltage of motor with an angle  $\cos^{-1} 0.8$ .

$$\therefore V_{tm} = 0.9697 \angle 0^\circ \text{ p.u.}$$

$$\text{and } I_L = 0.8594 \angle \cos^{-1} 0.8 = 0.8594 \angle 36.9^\circ \text{ p.u.}$$



## Method-1- applying KVL

### Prefault condition

The voltages and currents in the various elements of the system just before the fault are shown in fig (3). In this circuit Prefault voltage at fault point 'F' is  $V_{tm}$  and Load current  $I_L$  are known values and using these values the subtransient internal voltages of Synchronous generator  $E_g''$  and Synchronous Motor  $E_m''$  can be calculated by Kirchoff's Voltage Law (KVL) as shown below.

By applying KVL in the circuit of fig (3) we get,

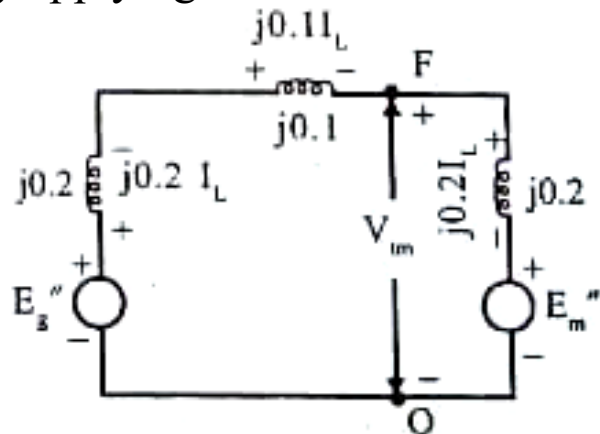


Fig 3 : Prefault current and voltages

$$\begin{aligned}
 E_g'' &= j0.2 I_L + j0.1 I_L + V_{tm} \quad \text{In polar form, } j = 1 \angle 90^\circ \\
 &= j0.3 I_L + V_{tm} \\
 &= 0.3 \angle 90^\circ \times 0.8594 \angle 36.9^\circ + 0.9697 \angle 0^\circ \\
 &= 0.2578 \angle 126.9^\circ + 0.9697 \angle 0^\circ \\
 &= -0.1548 + j0.2062 + 0.9697 \\
 &= 0.8149 + j0.2062 = 0.8406 \angle 14.2^\circ \text{ p.u.}
 \end{aligned}$$

$$\begin{aligned}
 E_m'' + j0.2 I_L &= V_{tm} \\
 \therefore E_m'' &= V_{tm} - j0.2 I_L = 0.9697 \angle 0^\circ - (0.2 \angle 90^\circ \times 0.8594 \angle 36.9^\circ) \\
 &= 0.9697 \angle 0^\circ - (0.1719 \angle 126.9^\circ) = 0.9697 - (-0.1032 + j0.1375) \\
 &= 1.0729 - j0.1375 = 1.0817 \angle -7.3^\circ \text{ p.u.}
 \end{aligned}$$

## Fault condition

The voltages and currents in the various elements of the system during subtransient state of the fault condition are shown in fig 4.

The circuit of fig.4. can be treated as two parallel circuits feeding current in the fault point.

Using KVL in the circuit of fig 4. we get

$$j0.2 I_g'' + j0.1 I_g'' = E_g'' \quad \text{or} \quad j0.3 I_g'' = E_g''$$

$$\therefore \text{Subtransient fault current in generator} \left\{ I_g'' = \frac{E_g''}{j0.3} = \frac{0.8406 \angle 14.2^\circ}{0.3 \angle 90^\circ} = 2.802 \angle -75.8^\circ \text{ p.u.} \right.$$

Using KVL in the circuit of fig 3.1.4. we get,  $j0.2 I_m'' = E_m''$

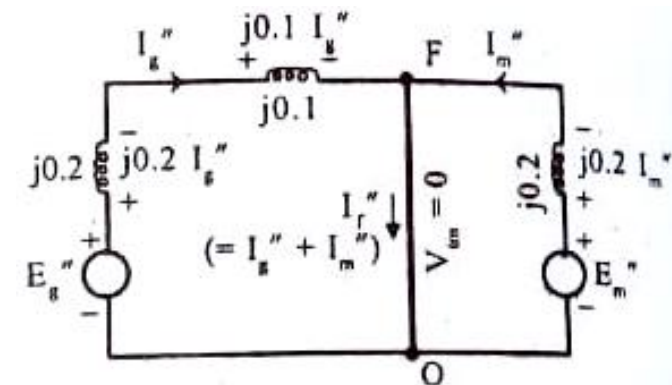
$$\therefore \text{Subtransient fault current in motor} \left\{ I_m'' = \frac{E_m''}{j0.2} = \frac{1.0817 \angle -7.3^\circ}{0.2 \angle 90^\circ} = 5.4085 \angle -97.3^\circ \text{ p.u.} \right.$$

$$\text{The current in the fault during subtransient state} \left\{ I_f'' = I_g'' + I_m'' \right.$$

$$= 2.802 \angle -75.8^\circ + 5.4085 \angle -97.3^\circ$$

$$= 0.687 - j2.716 - 0.687 - j5.365$$

$$= -j8.081 = 8.081 \angle -90^\circ \text{ p.u.}$$



**Fig 4 : Subtransient current and voltages**



The actual value of fault current can be obtained by multiplying the p.u. values of currents with base current.

$$\left. \begin{array}{l} \text{Actual value of subtransient} \\ \text{fault current in generator} \end{array} \right\} I_g^* = 2.802 \angle -75.8^\circ \times 1312.16$$

$$= 3676.67 \angle -75.8^\circ \text{ A} = 3.67667 \angle -75.8^\circ \text{ kA}$$

$$\left. \begin{array}{l} \text{Actual value of subtransient} \\ \text{fault current in motor} \end{array} \right\} I_m^* = 5.4085 \angle -97.3^\circ \times 1312.16$$

$$= 7096.8 \angle -97.3^\circ \text{ A} = 7.0968 \angle -97.3^\circ \text{ kA}$$

$$\left. \begin{array}{l} \text{Actual value of current in the} \\ \text{fault during subtransient state} \end{array} \right\} I_f^* = 8.081 \angle -90^\circ \times 1312.16$$

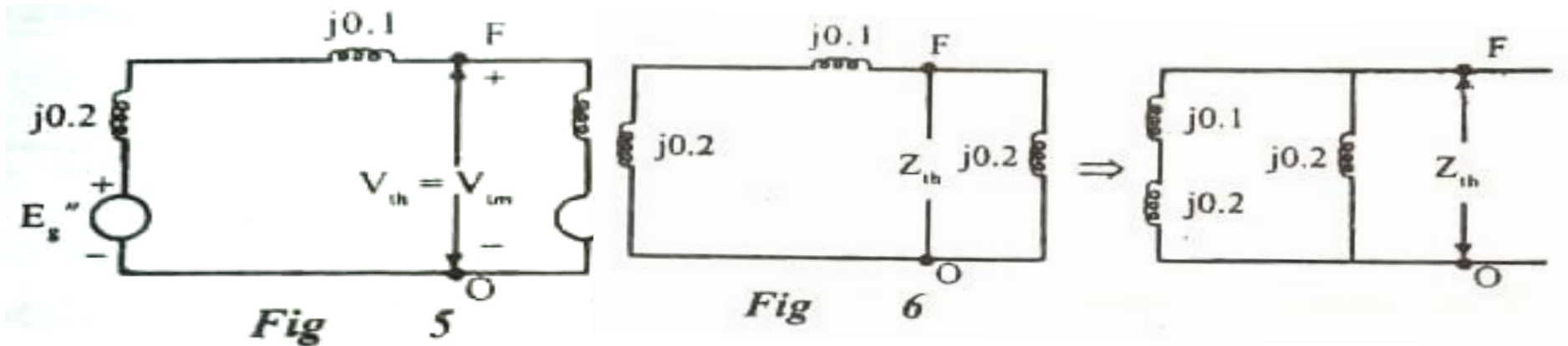
$$= 10603.56 \angle -90^\circ \text{ A} = 10.60356 \angle -90^\circ \text{ kA}$$

## Method-2 Using thevenin's theorem

### To find fault current

Since the prefault voltage at the fault point is known, the prefault circuit model can be represented by the thevenin's equivalent circuit.

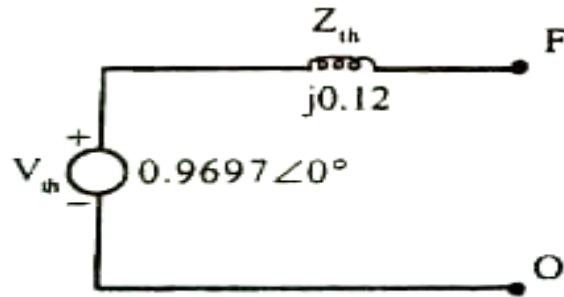
Consider the prefault circuit model shown in fig 5. The thevenin's voltage at the fault point is  $V_{tm}$ . The thevenin's impedance is given by the looking back impedance from the fault point. It is obtained by replacing all the voltage sources by zero value sources (i.e., by short circuit) as shown in fig 6 and then reducing the resultant network to single equivalent impedance as shown below.



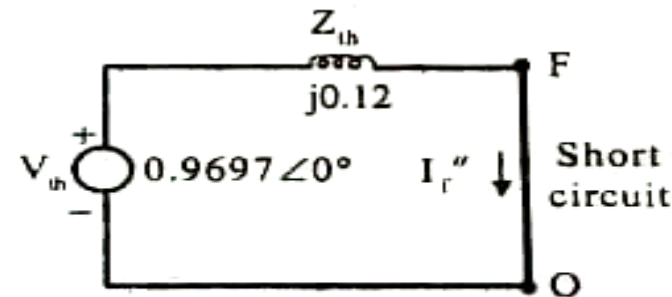
Thevenin's equivalent impedance

$$Z_{th} = \frac{(j0.1 + j0.2) j0.2}{(j0.1 + j0.2) + j0.2} = j0.12$$

The thevenin's equivalent of the circuit with respect to fault point is shown in fig 7. Now short circuiting the terminals of the thevenin's equivalent circuit as shown in fig .8. is equivalent to fault condition. The current flowing through the short is the fault current



**Fig 7 :** Prefault thevenin's equivalent at fault point

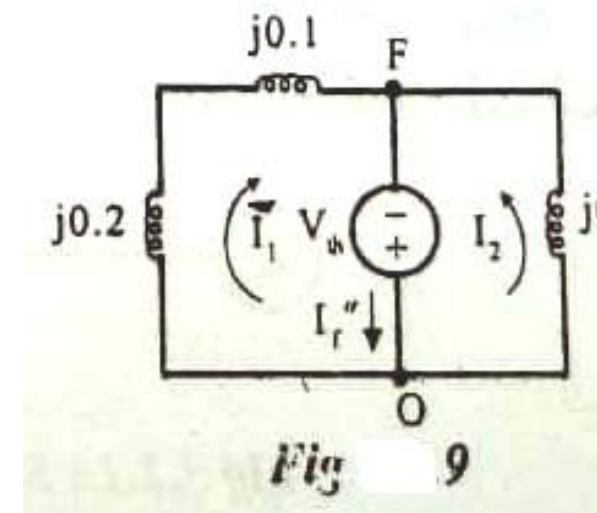


**Fig 8 :** Thevenin's equivalent under fault condition

$$\text{Current in the fault, } I_f'' = \frac{V_{th}}{Z_{th}} = \frac{0.9697 \angle 0^\circ}{j0.12} = \frac{0.9697 \angle 0^\circ}{0.12 \angle 90^\circ} = 8.081 \angle -90^\circ \text{ p.u.}$$

## To Find The change in current due to fault

The change in current due to fault can be calculated by connecting the thevenin's generator with reversed polarity at the fault point F as shown in fig .9. (Here all other sources are replaced by zero value sources, i.e., the voltage sources are replaced by short circuit).



$$\text{Now, } I_1 = \frac{V_{th}}{j0.2 + j0.1} = \frac{0.9697 \angle 0^\circ}{0.3 \angle 90^\circ} = 3.2323 \angle -90^\circ$$

$$I_2 = \frac{V_{th}}{j0.2} = \frac{0.9697 \angle 0^\circ}{0.2 \angle 90^\circ} = 4.8485 \angle -90^\circ$$

## To find the subtransient fault current in motor and generator

The subtransient fault currents of motor and generator are given by the sum of prefault current and the change in current due to fault (current delivered by thevenin's generator). Here the prefault current is the load current,  $I_L$ .

$$\begin{aligned} \therefore I_g'' &= I_1 + I_L = 3.2323 \angle -90^\circ + 0.8594 \angle 36.9^\circ = -j3.2323 + 0.6872 + j0.516 \\ &= 0.6872 - j2.7163 = 2.802 \angle -75.8^\circ \text{ p.u.} \end{aligned}$$

$$\begin{aligned} \therefore I_m'' &= I_2 - I_L = 4.8485 \angle -90^\circ - (0.8594 \angle 36.9^\circ) = -j4.8485 - (0.6872 + j0.516) \\ &= -0.6872 - j5.3645 = 5.4083 \angle -97.3^\circ \text{ p.u.} \end{aligned}$$

*Note :* It can be observed that the currents calculated by both the method are same.

## RESULT

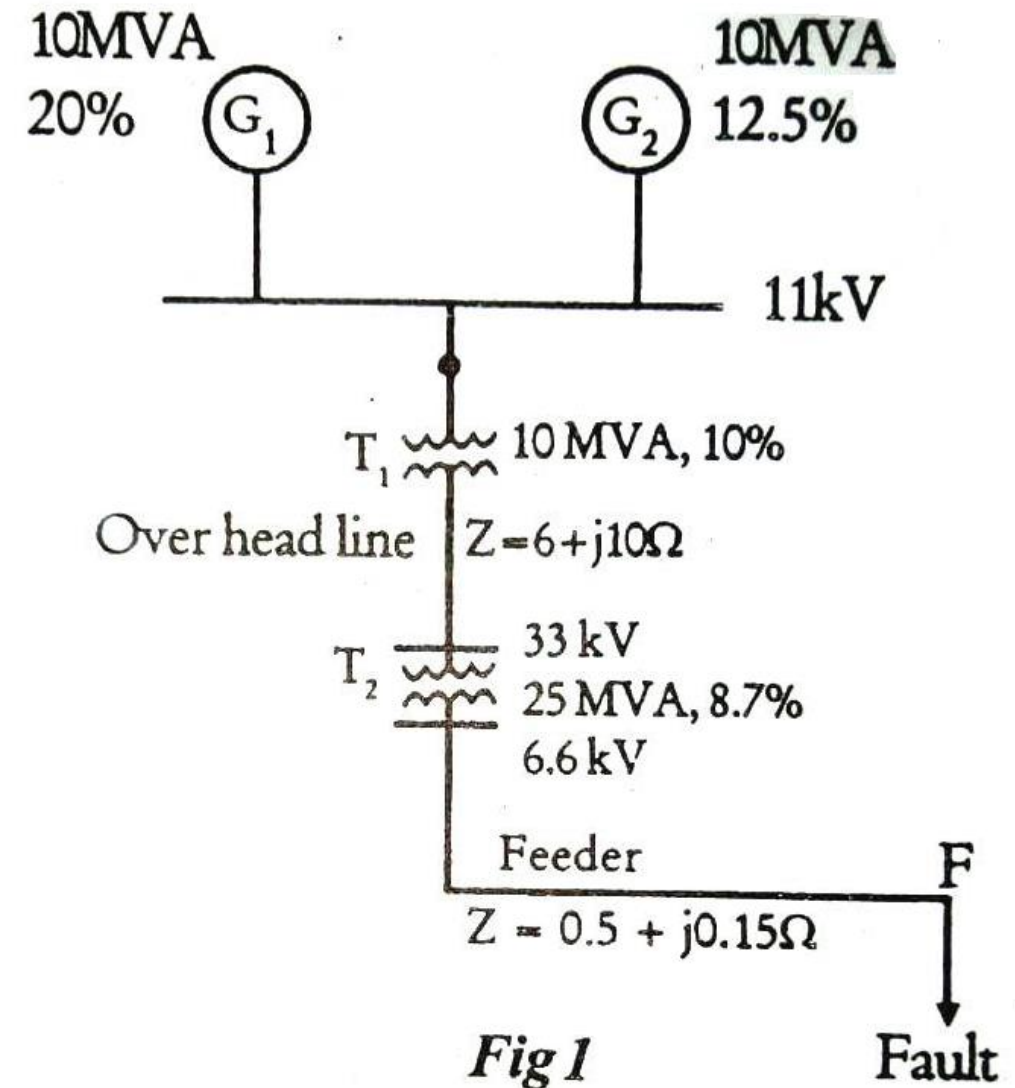
Subtransient fault current in generator,  $I_g'' = 2.802 \angle -75.8^\circ \text{ p.u.}$  or  $3.67667 \angle -75.8^\circ \text{ kA}$

Subtransient fault current in motor,  $I_m'' = 5.4085 \angle -97.3^\circ \text{ p.u.}$  or  $7.0992 \angle -97.3^\circ \text{ kA}$

Subtransient current in the fault,  $I_f'' = 8.081 \angle -90^\circ \text{ p.u.}$  or  $10.60356 \angle -90^\circ \text{ kA}$



**Numerical 3.** For the radial network shown in below Fig 1, a three-phase fault occurs at F. Determine the fault current and the line voltage at 11kV bus under fault conditions.



## Prefault Impedance diagram

$X_{d1}$ ,  $X_{d2}$  - Reactances of generator 1&2

$X_{T1}$ - Reactance of transformer T1.

$Z_{TL}$  - Impedance of transmission line.

$X_{T2}$ - Reactance of transformer T2.

$Z_{feed}$ - Impedance of feeder.

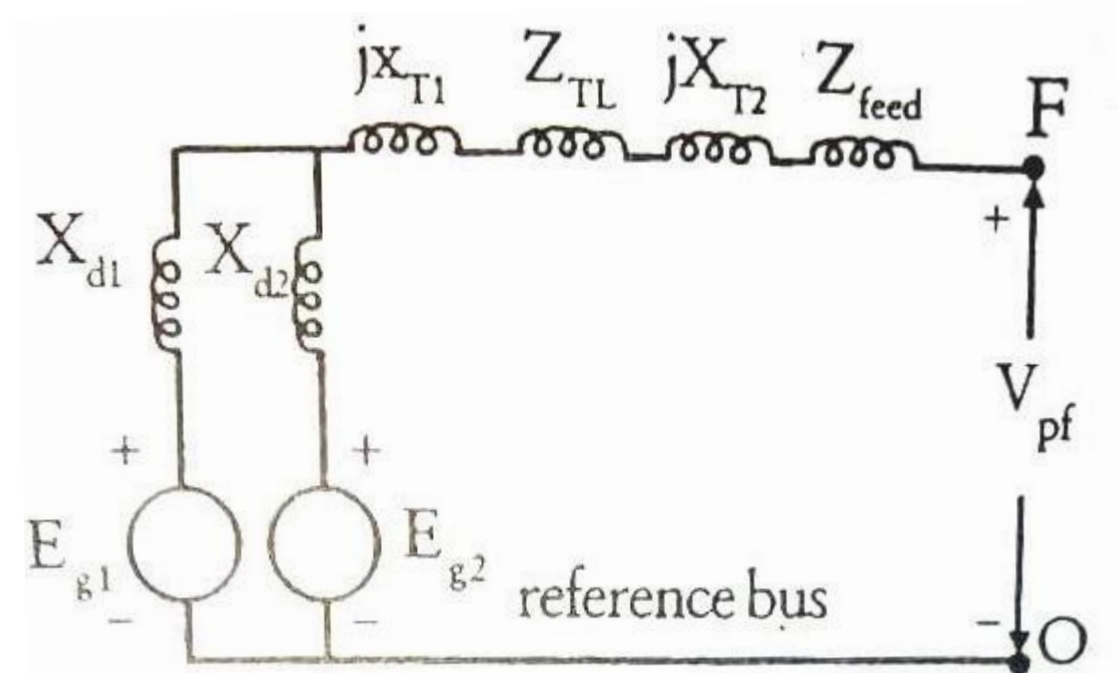


Fig.2

Let us choose the generator ratings as base values  $MVA_b = 10MVA$ ,  $kV_b = 11kV$

There is no load on the system and so the induced emf of the generator can be assumed as 1 p.u. Therefore the prefault voltage  $V_{pf}$  at the fault point is also 1 P.U

### Generator reactances

Since the generator ratings are chosen as base values, the p.u. reactance of the will remain same

p. u. reactance of generator-1,  $X_d = 20\% = 0.2$  p.u,

p.u. reactance of generator-2,  $X_d = 12.5\% = 0.125$  p.u

### Reactance of T1

The base values referred to LT side of transformer is same as chosen base and so its reactance is same as specified value.

p.u. reactance of transformer-T1,  $X_{T1} = 10\% = 0.1$  p.u.



### p.u. impedance of overhead line

$$\text{Base kV on HV side} = \text{Base kV on LV side} \times \frac{\text{HV rating}}{\text{LV rating}}$$

$$\text{Base kV on HT side of Transformer - T1} = 11 \times 33/11 = 33\text{kV}$$

$$\text{Base impedance, } Z_b = \frac{(\text{kV})^2}{\text{MVA}}$$

$$\text{Base impedance } Z_b = (33)^2 / 10 = 108.9 \text{ ohm/phase}$$

Given that the actual impedance of overhead line =  $6+j10$  ohm

$$\text{Actual impedance } 6+j10 \text{ p.u} \quad \text{Base impedance } 108.9$$

$$\begin{aligned} \text{p.u impedance overhead line, } Z_{TL} &= \text{Actual impedance} / \text{Base impedance} \\ &= 6+j10 / 108.9 = 0.0551 + j0.0918 \text{ p.u} \end{aligned}$$

## Reactance of T<sub>2</sub>

$$\text{New p.u. reactance} = X_{\text{pu,old}} \times \left[ \frac{\text{kV}_{\text{b,old}}}{\text{kV}_{\text{b,new}}} \right]^2 \times \frac{\text{MVA}_{\text{b,new}}}{\text{MVA}_{\text{b,old}}}$$

p.u. reactance of transformer-T<sub>2</sub>,  $X_{T2} = 0.087 \times (33/33)^2 \times (10/25) = 0.0348 \text{ p.u.}$

## P.U Impedance of feeder

$$\text{Base kV on LV side} = \text{Base kV on HV side} \times \frac{\text{LV rating}}{\text{HV rating}}$$

$$\text{Base kV on LT side of Transformer -T}_2 = 33 \times 11/33 = 11\text{kV}$$

$$\text{Base impedance, } Z_b = \frac{(\text{kV})^2}{\text{MVA}}$$

$$\text{Base impedance } Z_b = (6.6)^2 / 10 = 4.356 \text{ ohm/phase}$$

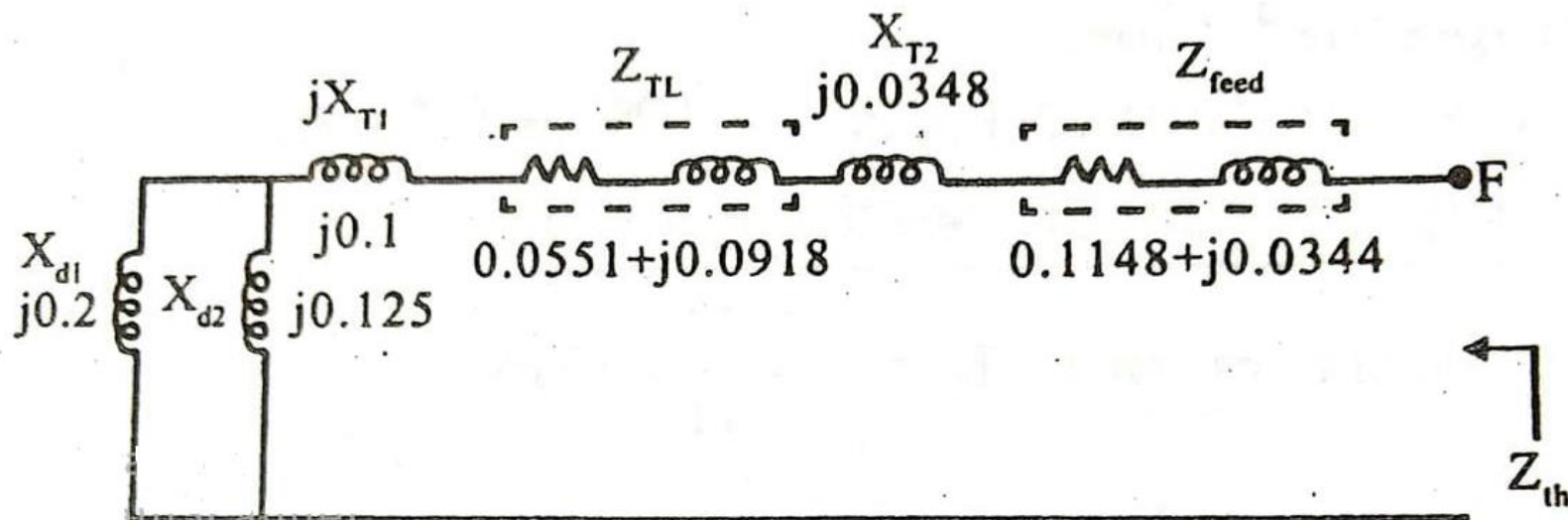
$$\begin{aligned} \text{p.u impedance feeder } Z_{\text{feed}} &= \text{Actual impedance} / \text{Base impedance} \\ &= 0.5 + j0.15 / 4.356 = 0.1148 + j0.0344 \text{ p.u} \end{aligned}$$

To find thevenin's equivalent at fault point

The thevenin's equivalent network of the prefault impedance diagram shown in below fig 2. can be obtained as shown below.

The thevenin's voltage at the fault point is the prefault voltage and so  $V = 1 \angle 0^\circ$  p.u

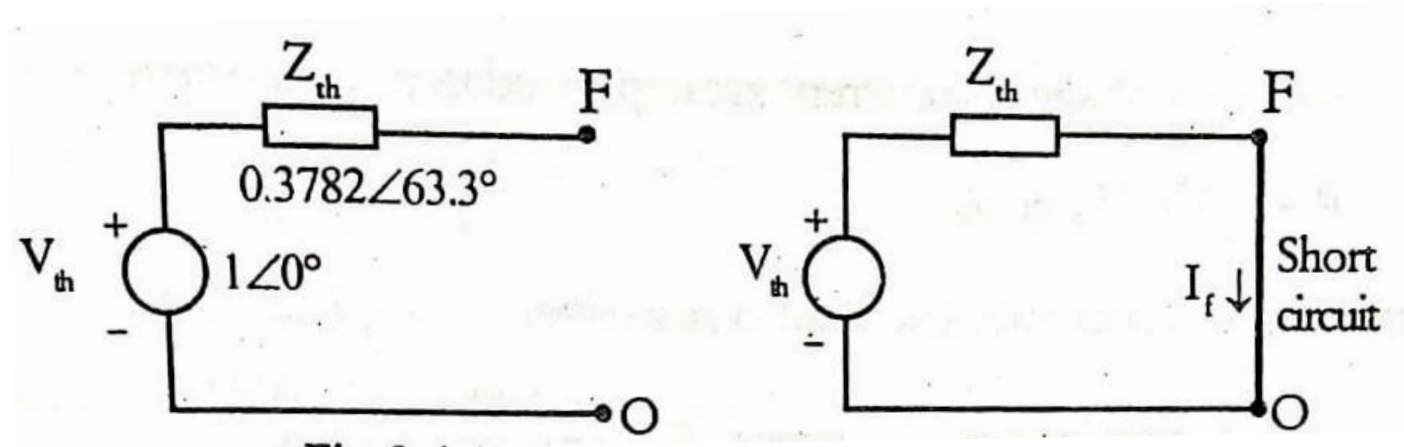
The thevenin's impedance can be obtained by reducing the following network (below fig 3.) to single equivalent impedance.



$$Z_{th} = \frac{j0.2 \times j0.125}{j0.2 + j0.125} + (j0.1 + 0.0551 + j0.0918 + j0.0348 + 0.1148 + j0.0344)$$

$$= j0.0769 + 0.1699 + j0.261 = 0.1699 + j0.3379 = 0.3782 \angle 63.3^\circ \text{ p.u.}$$

The prefault thevenin's equivalent circuit is shown in below fig. In this the fault is represented by a short circuit as shown in below fig. The current through the short circuit is the fault current  $I_f$



To find fault current

The P.u value of the fault current  $I_f = \frac{V_{th}}{Z_{th}} = \frac{1 \angle 0^\circ}{0.3782 \angle 63.3^\circ} = 2.6441 \angle -63.3^\circ \text{ p.u.}$

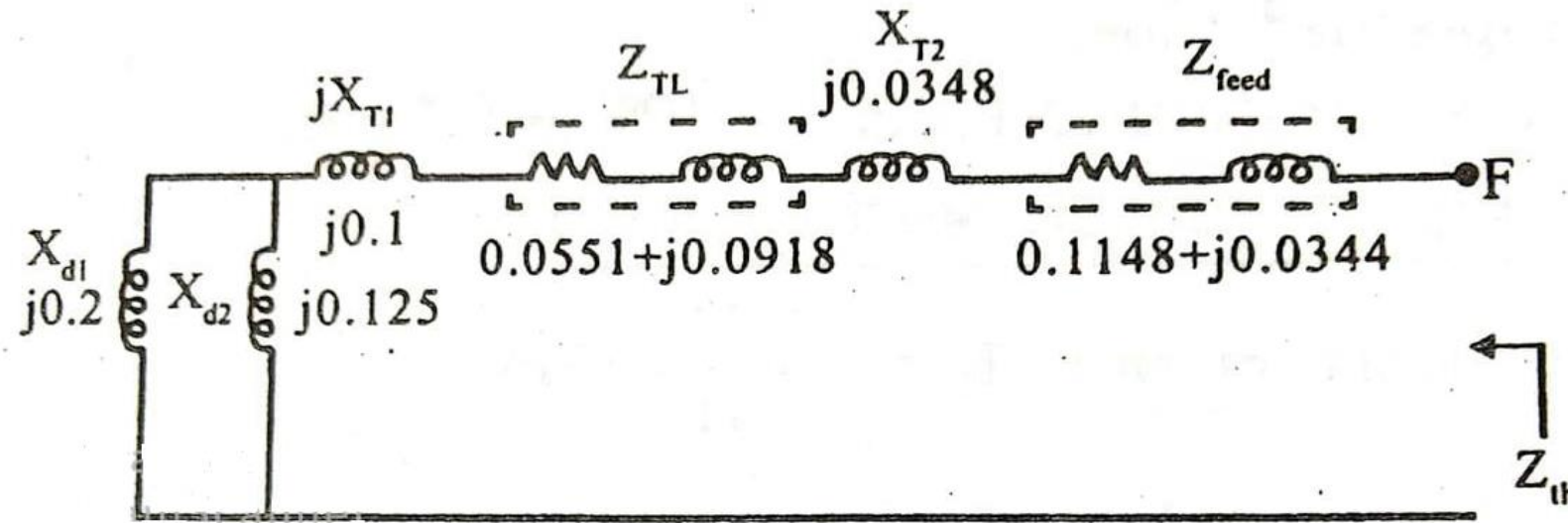
The base current  $I_b = \frac{kVA_b}{\sqrt{3}kV_b} = \frac{10 \times 1000}{\sqrt{3} \times 6.6} = 874.77 \text{ A}$

The Actual Value of fault current  $I_f = \text{p.u value of fault current} * \text{Base current}$

$$= 2.6441 \angle -63.3^\circ \times 874.77 = 2313 \angle -63.3^\circ \text{ A}$$

$$= 2.313 \angle -63.3^\circ \text{ kA}$$

The Fault current  $I_f = 2.6441 \angle -63.3^\circ \text{ p.u. or } 2.313 \angle -63.3^\circ \text{ kA}$



To find Voltage at 11kV Bus during fault

Find the total impedance between the Fault Point F and 11kV bus

$$Z' = 0.2699 + j0.161$$

$$\text{Voltage at 11kV bus} = [\text{Fault current } I_f * Z'] * V_b \text{ at 11kV}$$



## Numerical 3

A generator is connected through a transformer to a synchronous motor. The subtransient reactances of generator and motor are 0.15 and 0.35 respectively. The leakage reactance of the transformer is 0.1 p.u. All the reactances are calculated on a common base. A three phase fault occurs at the terminals of the motor when the terminal voltage of the generator is 0.9 p.u. The output current of generator is 1 p.u. and 0.8 pf leading. Find the subtransient current in p.u. in the fault, generator and motor. Use the terminal voltage of generator as reference vector.

The single line diagram and the circuit model to compute subtransient fault current are shown in Fig 1 and 2. respectively

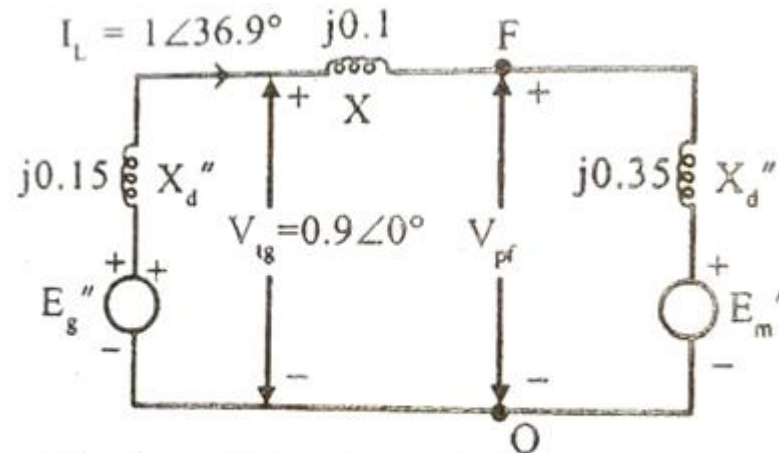
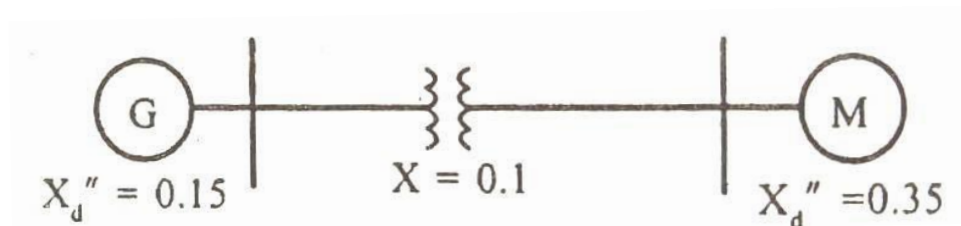


Fig 2 : Circuit model to compute subtransient fault current



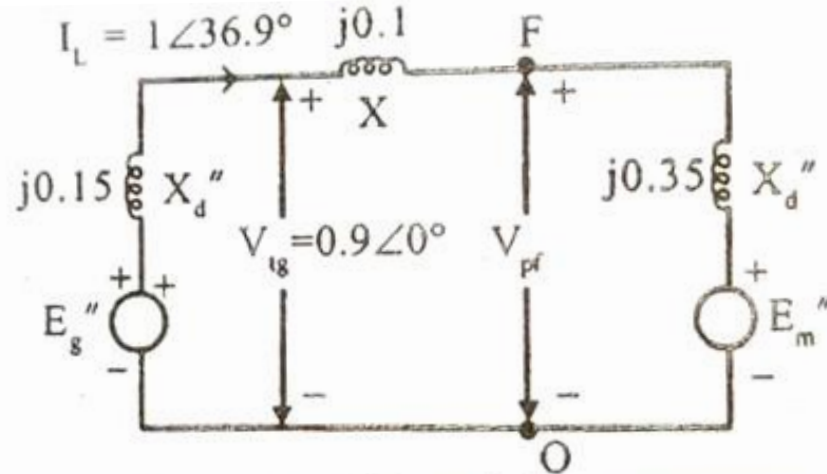


Fig 2 : Circuit model to compute subtransient fault current

In Fig 2. the generator and motor are represented by a source in series with subtransient reactance. The value of the sources are subtransient internal voltages

$V_{tg}$  is the Terminal voltage of generator

$V_{pf}$  is the prefault voltage at the fault point

The terminal voltage of generator as reference vector and so the load current will lead the generator terminal voltage by an angle  $\cos^{-1} 0.8 = 36.9^\circ$

Therefore load current in P.u  $= 1 \angle \cos^{-1} 0.8 = 1 \angle 36.9^\circ$  P.U

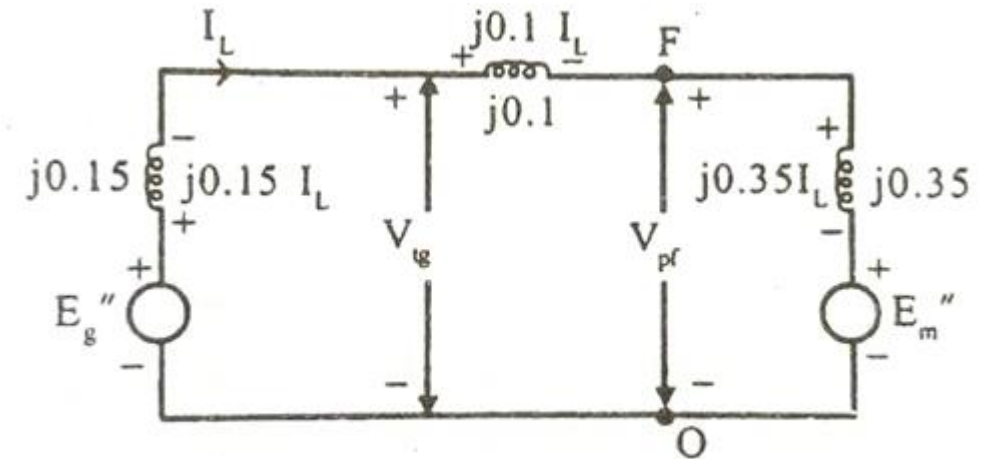
## Method 1-Symmetrical fault current estimation using Kirchoff's laws

### Prefault Conditions

The voltages and currents in the various elements of the system just before the fault are shown in fig (3). In this circuit  $V_{tg}$  and Load current  $I_L$  are known values and using these values the subtransient internal voltages of Synchronous generator  $E_g''$  and Synchronous Motor  $E_m''$  can be calculated by Kirchoff's Voltage Law (KVL) as shown below.

By applying KVL in the circuit of fig 3 we get,

$$\begin{aligned}
 E_g'' &= j0.15 I_L + V_{tg} \\
 &= 0.15 \angle 90^\circ \times 1 \angle 36.9^\circ + 0.9 \angle 0^\circ \\
 &= 0.15 \angle 126.9^\circ + 0.9 \angle 0^\circ = -0.0901 + j0.12 + 0.9 \\
 &= 0.8099 + j0.12 = 0.8187 \angle 8.4^\circ \text{ p.u.}
 \end{aligned}$$



*Fig 3 : Prefault current and voltages*

Using Kirchoff's Voltage Law (KVL) in the circuit of Fig 3 we get.

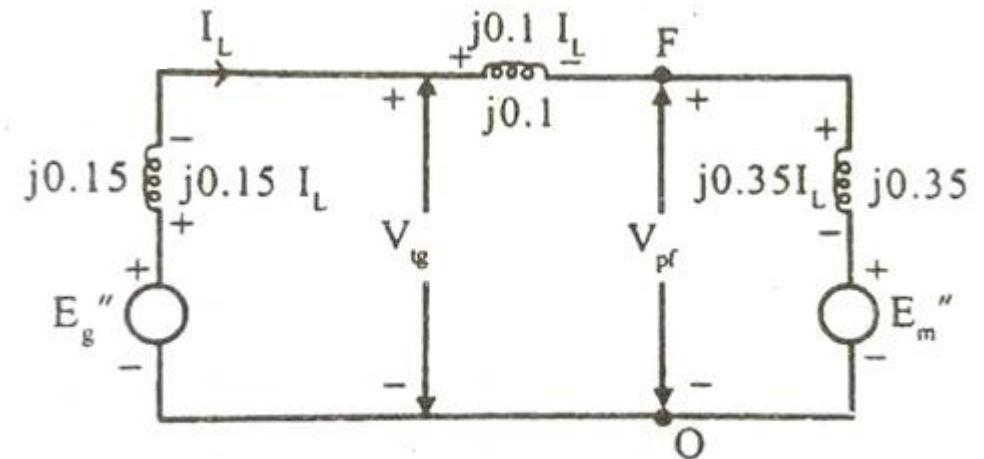
$$E_m'' + j0.35 I_L + j0.1 I_L = V_{tg}$$

$$\therefore E_m'' = V_{tg} - j0.45 I_L$$

$$= 0.9 \angle 0^\circ - (0.45 \angle 90^\circ \times 1 \angle 36.9^\circ) = 0.9 \angle 0^\circ - (0.45 \angle 126.9^\circ)$$

$$= 0.9 - (-0.2702 + j0.3599) = 1.1702 - j0.3599$$

$$= 1.2243 \angle -17.1^\circ \text{ p.u.}$$



*Fig 3 : Prefault current and voltages*

## Fault condition

The voltages and currents in the various elements of the system during subtransient state of the fault condition are shown in fig 4.  
The circuit of fig.4. can be treated as two parallel circuits feeding current in the fault point

Using Kirchoff's Voltage Law (KVL)

$$j0.15 I_g'' + 0.1 I_g'' = E_g'' \quad \text{or} \quad j0.25 I_g'' = E_g''$$

$$\therefore \text{Subtransient fault current in generator} \left\{ I_g'' = \frac{E_g''}{j0.25} = \frac{0.8187 \angle 8.4^\circ}{0.25 \angle 90^\circ} = 3.2748 \angle -81.6^\circ \text{ p.u.} \right.$$

Using KVL in the circuit of fig 4 we get,

$$j0.35 I_m'' = E_m''$$

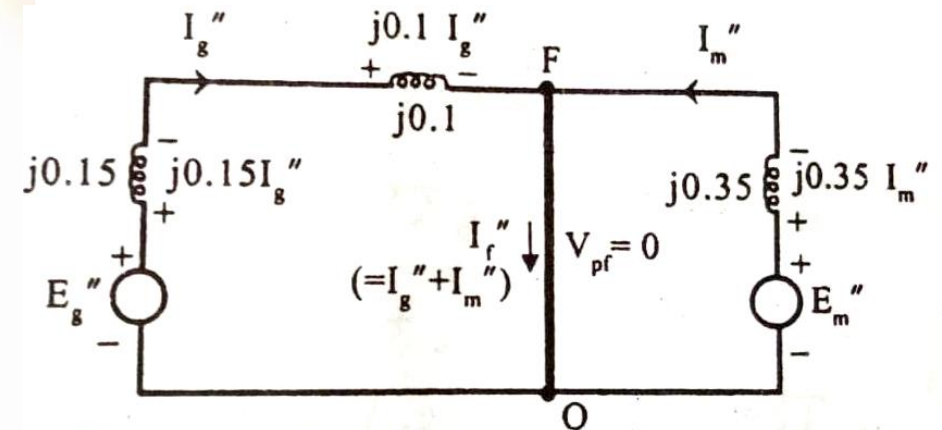
$$\therefore \text{Subtransient fault current in motor, } I_m'' = \frac{E_m''}{j0.35} = \frac{1.2243 \angle -17.1^\circ}{0.35 \angle 90^\circ} = 3.498 \angle -107.1^\circ \text{ p.u.}$$

The current in the fault during subtransient state  $I_f'' = I_g'' + I_m''$

$$= 3.2748 \angle -81.6^\circ + 3.498 \angle -107.1^\circ$$

$$= 0.4784 - j3.2397 - 1.0286 - j3.3434$$

$$= -0.5502 - j6.5831 = 6.606 \angle -94.8^\circ \text{ p.u.}$$



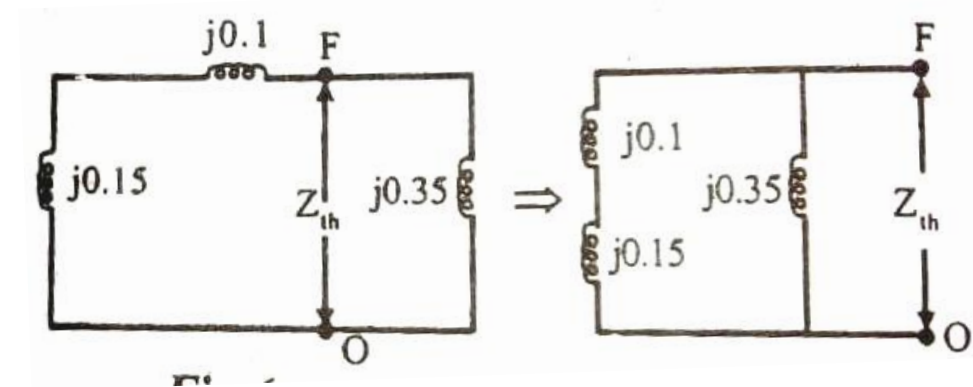
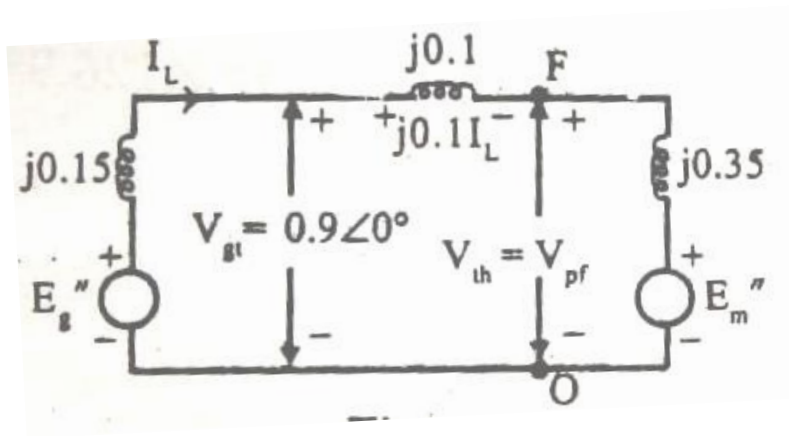
**Fig 4 : Subtransient currents and voltages**

## Method-2 Using thevenin's theorem

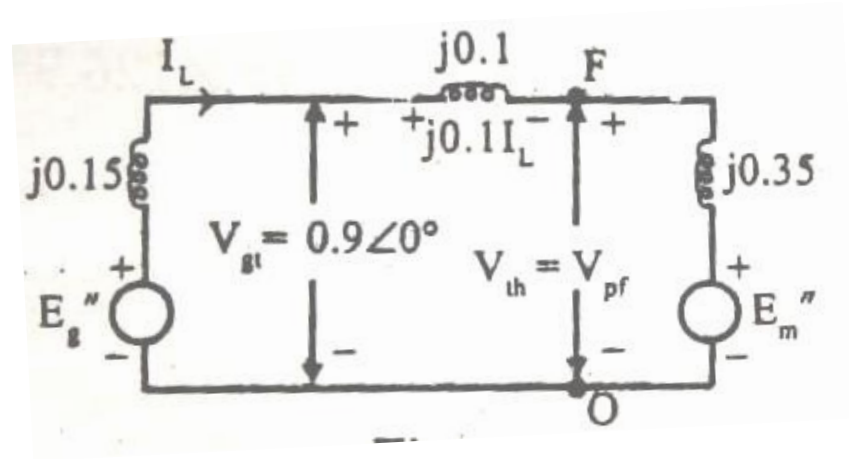
### To find fault current

Since the prefault voltage at the fault point can be estimated from the prefault circuit model. Then the thevenin's equivalent of the circuit is determined and from which the fault current is estimated

Consider the prefault circuit model shown in fig 5. The thevenin's voltage at the fault point 'F' is  $V_{pf}$ . The thevenin's impedance is given by the looking back impedance from the fault point 'F'. It is obtained by replacing all the voltage sources by zero value sources (i.e., by short circuit) as shown in fig 6 and then reducing the resultant network to single equivalent impedance as shown below.







Using Kirchoff's Voltage Law (KVL) in the above circuit of Fig 5 we get.

$$V_{gt} = j0.1 I_L + V_{th}$$

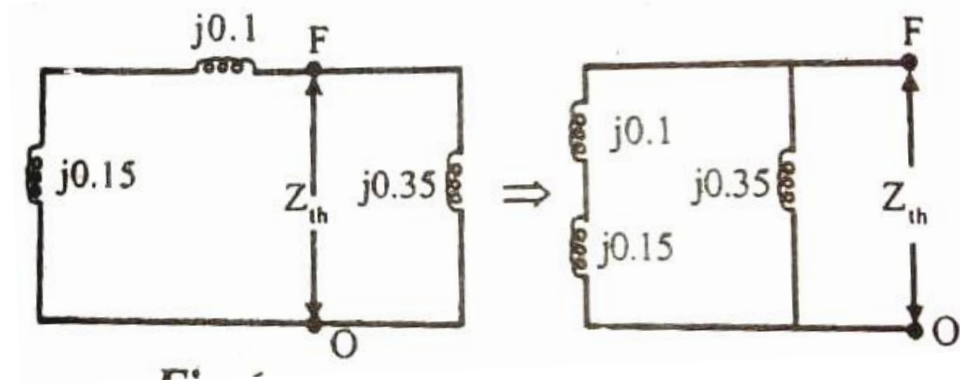
$$\text{Thevenin's voltage, } V_{th} = V_{gt} - j0.1 I_L$$

$$= 0.9 \angle 0^\circ - (0.1 \angle 90^\circ \times 1 \angle 36.9^\circ)$$

$$= 0.9 \angle 0^\circ - (0.1 \angle 126.9^\circ)$$

$$= 0.9 - (-0.06 + j0.08)$$

$$= 0.96 - j0.08 = 0.9633 \angle -4.8^\circ \text{ p.u.}$$

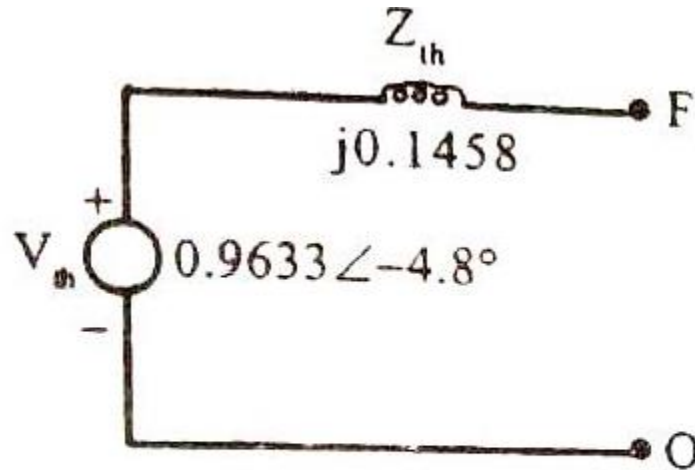


Thevenin's equivalent impedance  $Z_{th}$

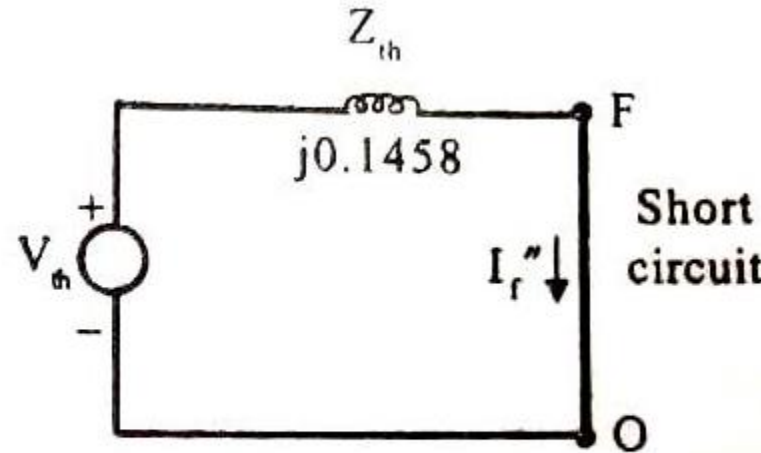
$$Z_{th} = \frac{(j0.15 + j0.1) j0.35}{(j0.15 + j0.1) + j0.35} = j0.1458 \text{ p.u.}$$



The thevenin's equivalent of the circuit with respect to fault point is shown in fig 7. Now short circuiting the terminals of the thevenin's equivalent circuit as shown in fig .8. is equivalent to fault condition. The current flowing through the short is the fault current



*Fig 7 : Prefault thevenin's equivalent at fault point*

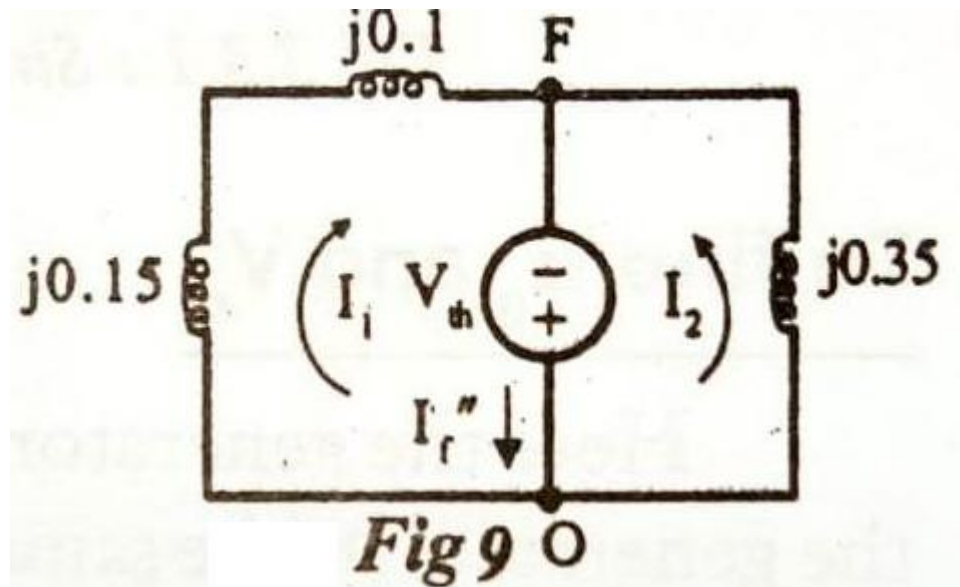


*Fig 8 : Thevenin's equivalent under fault condition*

$$\text{Current in the fault, } I_f'' = \frac{V_{th}}{Z_{th}} = \frac{0.9633 \angle -4.8^\circ}{j0.1458} = \frac{0.9633 \angle -4.8^\circ}{0.1458 \angle 90^\circ} = 6.606 \angle -94.8^\circ \text{ p.u.}$$

## To Find The change in current due to fault

The change in current due to fault can be calculated by connecting the thevenin's generator with reversed polarity at the fault point F as shown in fig .9. (Here all other sources are replaced by zero value sources, i.e., the voltage sources are replaced by short circuit).



$$I_1 = \frac{V_{th}}{j0.15 + j0.1} = \frac{0.9633 \angle -4.8^\circ}{0.25 \angle 90^\circ} = 3.8532 \angle -94.8^\circ$$

$$I_2 = \frac{V_{th}}{j0.35} = \frac{0.9633 \angle -4.8^\circ}{0.35 \angle 90^\circ} = 2.7523 \angle -94.8^\circ \text{ p.u.}$$

## To find the subtransient fault current in motor and generator

The subtransient fault currents of motor and generator are given by the sum of prefault current and change in current due to fault (current delivered by thevenin's generator). Here the prefault current is the load current  $I_L$

$$\begin{aligned}\therefore I_g'' &= I_1 + I_L = 3.8532 \angle -94.8^\circ + 1 \angle 36.9^\circ = -0.3224 - j3.8397 + 0.7997 + j0.6004 \\ &= 0.4773 - j3.2393 = 3.274 \angle -81.6^\circ \text{ p.u.}\end{aligned}$$

$$\begin{aligned}\therefore I_m'' &= I_2 - I_L = 2.7523 \angle -94.8^\circ - 1 \angle 36.9^\circ = -0.2303 - j2.7426 - (0.7997 + j0.6004) \\ &= -1.03 - j3.343 = 3.498 \angle -107.1^\circ \text{ p.u.}\end{aligned}$$

Subtransient fault current in generator,  $I_g'' = 3.274 \angle -81.6^\circ$  p.u.

Subtransient fault current in motor,  $I_m'' = 3.498 \angle -107.1^\circ$  p.u.

Subtransient current in the fault,  $I_f'' = 6.606 \angle -94.8^\circ$  p.u.

For the radial network shown in fig. 2.6, a three phase fault occurs at F. Determine the fault current and the line voltage at 11kV bus under fault conditions.

Solution:

Base values: Let us choose,

base MVA=10

base kV in the overhead line=33

base kV on the generator side=33×11/33= 11

base kV on the cable side=33×6.6/33= 6.6

Reactance of generator G1:

$$X_{G1, \text{ new }} = X_{G1, \text{ old }} \times \left( \frac{(MVA)_{B, \text{ new }}}{(MVA)_{B, \text{ old }}} \right) \times \left( \frac{(kV)^2_{B, \text{ old }}}{(kV)^2_{B, \text{ new }}} \right)$$

$$= j0.15 \times (100 / 10) \times (11 / 11)^2 = j 1.5 \text{ p.u}$$

Reactance of generator G2:

$$X_{G2, \text{ new }} = X_{G2, \text{ old }} \times \left( \frac{(MVA)_{B, \text{ new }}}{(MVA)_{B, \text{ old }}} \right) \times \left( \frac{(kV)^2_{B, \text{ old }}}{(kV)^2_{B, \text{ new }}} \right)$$

$$= j0.125 \times (10 / 10) \times (11 / 11)^2$$

$$= j 1.25 \text{ p.u}$$

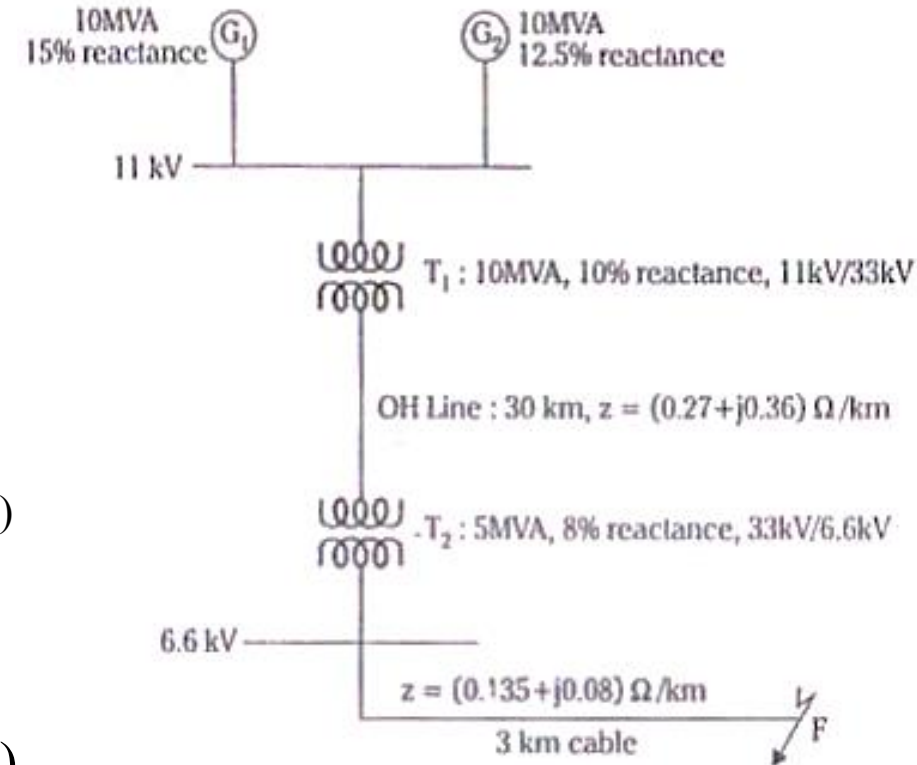


Fig. 2.6



Reactance of transformer T1: (calculated secondary side it)HV or HT

$$\begin{aligned} X_{T1, \text{ new}} &= X_{T1, \text{ old}} \times \left( \frac{(MVA)_{B, \text{ new}}}{(MVA)_{B, \text{ old}}} \right) \times \left( \frac{(kV)^2_{B, \text{ old}}}{(kV)^2_{B, \text{ new}}} \right) \\ &= j0.1 \times (100 / 10) \times (33 / 33)^2 \\ &= j 1.0 \text{ p.u} \end{aligned}$$

Reactance of transformer T2: (calculated primary side of it)HV or HT

$$\begin{aligned} X_{T2, \text{ new}} &= X_{T2, \text{ old}} \times \left( \frac{(MVA)_{B, \text{ new}}}{(MVA)_{B, \text{ old}}} \right) \times \left( \frac{(kV)^2_{B, \text{ old}}}{(kV)^2_{B, \text{ new}}} \right) \\ &= j0.08 \times (100 / 5) \times (33 / 33)^2 \\ &= j 1.6 \text{ p.u} \end{aligned}$$

Impedance of O.H line:

$$\begin{aligned} Z_{O.H, \text{ p.u}} &= Z_{O.H} (\Omega) \times (MVA)_{B, \text{ new}} / (kV)^2_{B, \text{ new}} \\ &= (30 \times (0.27 + j0.36)) \times 100 / 33^2 \\ &= 0.744 + j0.99 \text{ p.u} \end{aligned}$$

Impedance of cable:

$$\begin{aligned} Z_c &= Z_c (\Omega) \times (MVA)_{B, \text{ new}} / (kV)^2_{B, \text{ new}} \\ &= (3 \times (0.135 + j0.08)) \times 100 / 6.6^2 \\ &= 0.93 + j0.55 \text{ p.u} \end{aligned}$$

Shorting the generated voltages, we obtain the equivalent circuit of the system prior to the fault as in fig. 2.8



Fig. 2.8

$$\begin{aligned}
 Z_{TH} &= (j1.5 \times j1.25) / (j1.5 + j1.25) + (j1.0 + 0.744 + j0.99 + j1.6 + 0.93 + j0.55) \\
 &= 1.674 + j4.82 \\
 &= 5.1 \angle 70.8^\circ \text{ p.u.}
 \end{aligned}$$

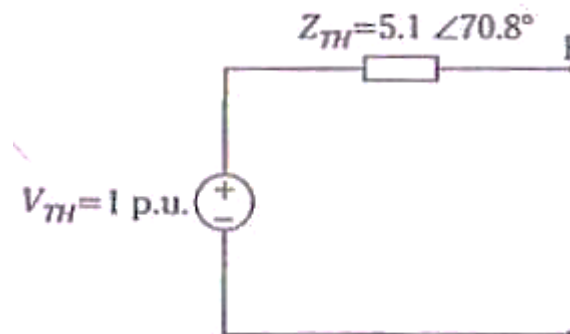


Fig. 2.9



Now short circuiting the terminals of the Thevenin's equivalent circuit as shown in fig. 2.10 is equivalent to the fault condition. The current flowing through the short circuit is the fault current.

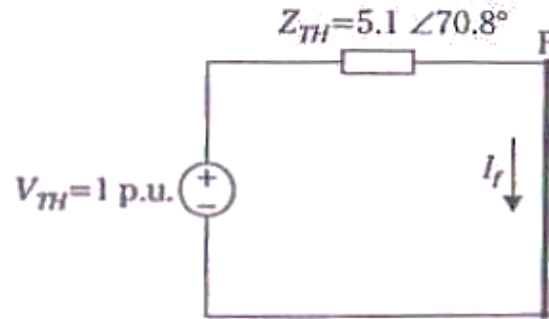


Fig. 2.10

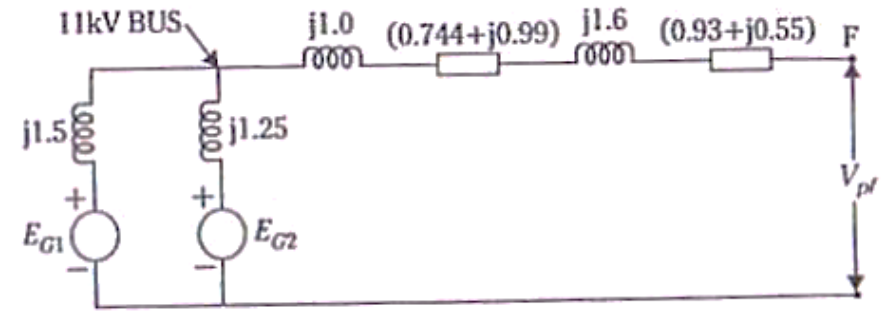


Fig. 2.7

The p.u value of fault current,  $I_f = V_{TH} / Z_{TH} = (1.0 \angle 0^\circ) / (5.1 \angle 70.8^\circ) = 0.196 \angle -70.8^\circ$  p.u

The base current,  $I_b = (1000 \times \text{base power}) / (\sqrt{3} \times \text{base voltage}) = (1000 \times 100) / (\sqrt{3} \times 6.6) = 8747$  A  
 therefore, absolute value of fault current,  $I_f = 0.196 \angle -70.8^\circ \times 8747 = 1714 \angle -70.8^\circ$  A

To find voltage at 11 kV bus during fault:

From fig 2.7, it can be observed that the total impedance between point F and 11kV bus is,  
 $= (0.93 + j0.55) + j1.6 + (0.744 + j0.99) + j1.0 = (1.674 + j4.14)$  p.u  $= 4.466 \angle 67.98^\circ$

Voltage at 11kV bus  $= 4.466 \angle 67.98^\circ \times 0.196 \angle -70.8^\circ = 0.875 \angle -2.82^\circ$  p.u

The absolute value of the voltage at 11kV bus  $= 0.875 \angle -2.82^\circ \times 11 = 0.9625 \angle -2.82^\circ$  kV

Two generators are connected in parallel to the low-voltage(L.V) side of a three phase  $\Delta$ -Y transformer. The ratings of the machines are

Generator G1: 50 MVA, 13.8kV,  $X_d''=25\%$

Generator G2: 25MVA, 13.8kV,  $X_d''=25\%$

Transformer T: 75MVA, 13.8  $\Delta$  -69 Y kV,  $X=10\%$

Before the fault occurs, the voltage on the high voltage (HV) side of the transformer is 66kV. The transformer is unloaded, and there is no circulating current between the generators. Find the subtransient current in each generator when a three phase fault occurs on the high voltage side of the transformer.

Solution:

base values: base MVA= 75, base kV on HV side of transformer=69 KV

base kV on the generator =  $69 \times 13.8 / 69 = 13.8$

Reactance of generator G1:

$$\begin{aligned} X_{G1, \text{ new}} &= X_{G1, \text{ old}} \times \left( \frac{(MVA)_B, \text{ new}}{(MVA)_B, \text{ old}} \right) \times \left( \frac{(kV)^2_B, \text{ old}}{(kV)^2_B, \text{ new}} \right) \\ &= j0.25 \times (75 / 50) \times (13.8 / 13.8)^2 = j 0.375 \text{ p.u} \end{aligned}$$

Reactance of generator G2:

$$\begin{aligned} X_{G2, \text{ new}} &= X_{G2, \text{ old}} \times \left( \frac{(MVA)_B, \text{ new}}{(MVA)_B, \text{ old}} \right) \times \left( \frac{(kV)^2_B, \text{ old}}{(kV)^2_B, \text{ new}} \right) \\ &= j0.25 \times (75 / 25) \times (13.8 / 13.8)^2 = j 0.75 \text{ p.u} \end{aligned}$$

Reactance of transformer:  $X_T = j0.1$

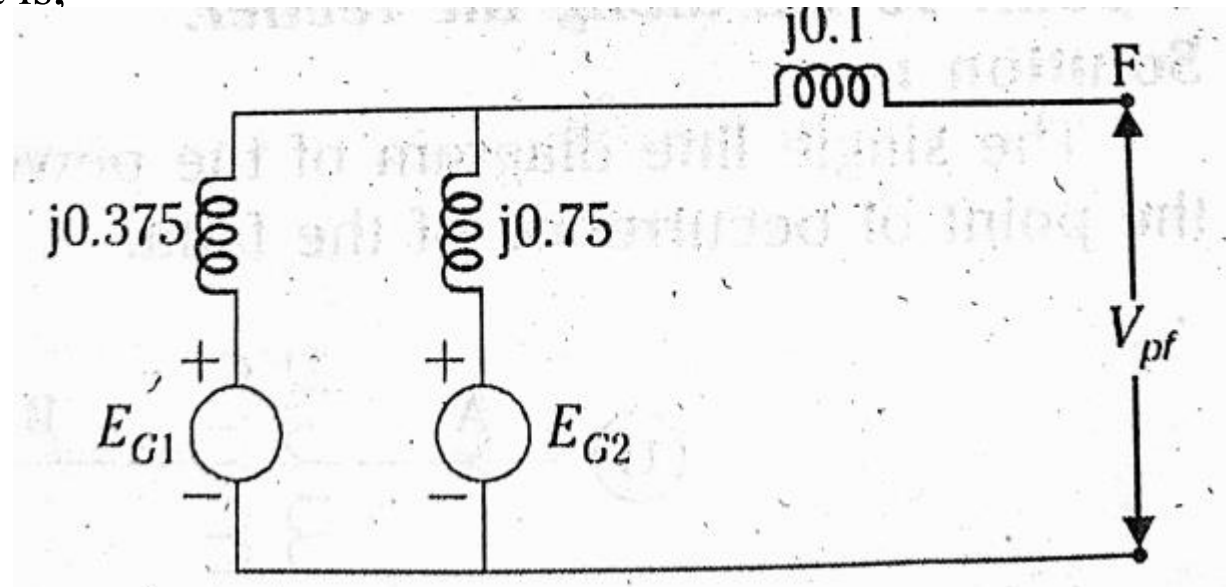
The reactance diagram of the system for the calculation of prefault values is as shown in fig.

The prefault voltage on the high voltage side is 66kV. This is equal to  $66/69=0.957$  p.u

The equivalent sub-transient reactance as visualised from the fault point is,  
 $((j0.375 \times j0.75)/(j0.375 + j0.75)) + j0.1 = j0.35$  p.u

therefore, the sub-transient current in the short circuit is,

$$I_f'' = 0.957 / j0.35 = 2.735 \angle -90^\circ \text{ p.u}$$



To find the sub-transient currents in the generators:

The sub-transient fault current divides between the generators inversely as the impedances of the generators.

In generator G1:

$$I_{g1}'' = 2.735 \angle -90^\circ \times (j0.75 / j1.125) = 1.823 \angle -90^\circ \text{ p.u}$$

In generator G2:

$$I_{g2}'' = 2.735 \angle -90^\circ \times (j0.375 / j1.125) = 0.912 \angle -90^\circ \text{ p.u}$$

The absolute values of the above currents can be obtained by multiplying the p.u values by the base current.

Base current,

$$I_B = (1000 \times \text{base power}) / (\sqrt{3} \times \text{base voltage}) = (1000 \times 75) / (\sqrt{3} \times 13.8) = \underline{3137.7 \text{ A}}$$

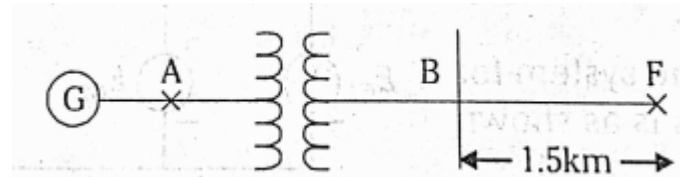
hence the actual currents are,

$$I_f'' = 2.735 \angle -90^\circ \times 3137.7 = 8581.6 \angle -90^\circ \text{ A}$$

$$I_{g1}'' = 1.823 \angle -90^\circ \times 3137.7 = 5720 \angle -90^\circ \text{ A}$$

$$I_{g2}'' = 0.912 \angle -90^\circ \times 3137.7 = 2861.6 \angle -90^\circ \text{ A}$$

A three - phase, 5 MVA, 6.6 kV alternator with reactance 8% is connected to a feeder of series impedance of  $(0.12 + j0.48)$  ohms/phase per Km. The transformer is rated at 3 MVA, 6.6 kV/33 kV and has a series reactance of 5%. Determine the fault current supplied by the generator operating under no - load with a voltage of 6.9 kV, when a three - phase symmetrical fault occurs at a point 15 km along the feeder.



### Base values

Let us choose the generator rating as base values.

$$\therefore (MVA)_B = 5 \text{ MVA.}$$

$$\therefore \text{Base voltage on the generator} = 6.6 \text{ kV.}$$

$$\text{Base voltage on the transmission line} = 6.6 \times \frac{33}{6.6} = 33 \text{ kV.}$$

### Reactance of generator

$$X_G = 8\% = j0.08 \text{ p.u.}$$

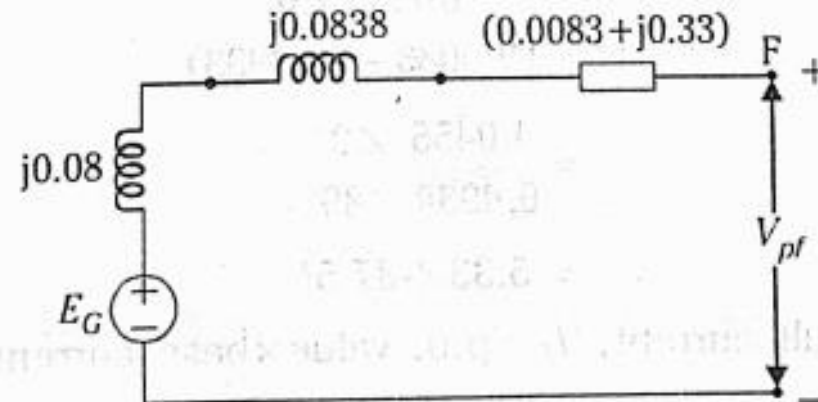
## Reactance of transformer

$$X_{T, new} = X_{T, old} \times \frac{(MVA)_{B, new}}{(MVA)_{B, old}} \times \frac{(kV)_{B, old}^2}{(kV)_{B, new}^2} = j0.05 \times \frac{(5)}{(3)} \times \frac{(33)^2}{(33)^2} = j0.833 \text{ p.u.}$$

## Impedance of the feeder

$$(Z_{TL})_{p.u.} = Z_{TL}(\Omega) \times \frac{(MVA)_B}{(kV)_B^2} = (0.12 + j0.48) \times 15 \times \frac{5}{(33)^2} = (0.0083 + j0.033) \text{ p.u.}$$

Using these values, the prefault impedance diagram is as shown in Fig.





To find  $E_G$  and  $V_{pf}$

Actual value of induced emf,  $E_G = 6.9$  kV.

$$\text{p.u. value of induced emf, } E_G = \frac{\text{Actual value}}{\text{Base value}} = \frac{6.9}{6.6}$$

$$= 1.0455 \text{ p.u.}$$

The prefault voltage,  $V_{pf}$  at fault point F is the voltage under no-load = 34.5 kV.

$\therefore$  Prefault voltage,  $V_{pf} = 34.5$  kV.

The p.u. value of prefault voltage,  $V_{pf} = \frac{\text{Actual value}}{\text{Base value}}$

$$= \frac{34.5}{33}$$

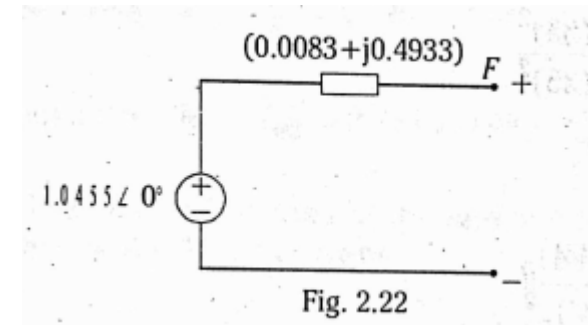
$$= 1.0455 \text{ p.u.}$$

To find fault current

The Thevenin's equivalent circuit of the system shown in Fig. 2.22 as seen from the fault point F is shown in Fig. 2.22. Here

$$V_{TH} = 1.045 \angle 0^\circ$$

$$Z_{TH} = j0.08 + j0.0833 + (0.0083 + j0.33) = 0.0083 + j0.4933$$





The fault in the feeder can be represented by a short circuit as shown in Fig. 2.23. Now the current  $I_f$  through the short circuit is the fault current.

$$\begin{aligned} \therefore \text{p.u. value of fault current, } I_f &= \frac{V_{TH}}{Z_{TH}} = \frac{1.0455 \angle 0^\circ}{(0.0083 + j0.4933)} = \frac{1.0455 \angle 0^\circ}{0.4934 \angle 89^\circ} \\ &= 5.33 \angle -87.5^\circ \end{aligned}$$

$\therefore$  Actual value of fault current,  $I_f = \text{p.u. value} \times \text{base current}$

$$\begin{aligned} &= 2.12 \angle -89^\circ \times \frac{5 \times 10^6}{\sqrt{3} \times 33 \times 10^3} \\ &= 466.25 \angle -87.5^\circ \text{ A.} \end{aligned}$$

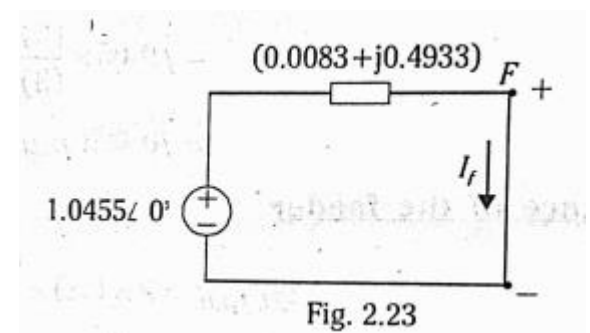


Fig. 2.23

## SELECTION OF CIRCUIT BREAKERS

The Two Function of Circuit breaker are

1. To act as a switch for normal load conditions
2. To automatically isolate or open the faulty part in the event of a fault.

Since the circuit breakers are employed at places where the power level is high, whenever its contacts open it has to interrupt heavy currents both during load conditions and faulty conditions.

Since the power system is predominantly inductive in nature, the interruption of current when the circuit break open its contact is associated with large voltages induced across its contacts which in turn results in sparking at the contacts. Hence in circuit breakers the amount of current it has to interrupt is an important criteria.

The circuit breaker for a particular application (or load) is selected based on the following ratings.

1. Normal working power level specified as rated interrupting current or rated interrupting kVA.
2. The fault level specified as either the rated short circuit interrupting current or rated short circuit interrupting MVA.
3. Momentary current rating.
4. Normal working voltage.
5. Speed of circuit breaker.

The speed of circuit breaker is the time between the occurrence of the fault to the extinction of the arc (when the contact opens).

It is normally specified in cycles of power frequency.[1 cycle for 50HZ power frequency is  $1/50 = 0.02$  sec or 20m sec].

**The standard speed of circuit breakers are 8, 5, 3 or 1½ cycles.**

The momentary current rating is the maximum current that may flow through a circuit breaker for a short duration. It is the current that may flow during subtransient period of fault condition.

In fault analysis the subtransient fault current calculated using subtransient circuit model is the symmetrical subtransient current. It is then multiplied by a factor of 1.6 to get maximum momentary current during fault, (The factor 1.6 accounts for dc-offset current during Subtransient period)

The circuit breaker is chosen such that its momentary current rating is less than the calculated value.

Usually the circuit breaker will open its contacts in the transient period and so the short circuit interrupting current rating depends on transient period currents.

In fault analysis the transient fault current calculated using transient circuit model is the symmetrical transient fault current. It is then multiplied by a factor 1.0 to 1.5 to get the maximum interrupting current.[The factor 1.0 to 1.5 accounts for dc-offset current during transient period].

The circuit breaker is chosen such that its short circuit interrupting current rating is less than the calculated value.

The multiplying factor to find interrupting current depends on the speed of circuit breaker.

The multiplying factor for various speeds of circuit breaker are shown in below table

Table : Multiplying factor to find the short circuit interrupting current

| Speed of circuit breaker | Multiplying factor |
|--------------------------|--------------------|
| 8 cycles or more         | 1.0                |
| 5 cycles                 | 1.1                |
| 3 cycles                 | 1.2                |
| 2 cycles                 | 1.4                |
| 1½ cycles                | 1.5                |



The short circuit interrupting MVA can be estimated from the knowledge of prefault voltage and short-circuit interrupting current as shown below.

$$\text{Short circuit interrupting MVA} = \sqrt{3} |V_{pfL}| |I_{fL}|$$

Where

$$|V_{pfL}| = \text{Magnitude of prefault line voltage at the fault point in kV.}$$

$$|I_{fL}| = \text{Magnitude of line value of short circuit interrupting current at the fault in kA}$$

$$\text{or Short circuit interrupting MVA} = |V_{pfL, pu}| \times |I_{fL, pu}| \times \text{MVA}_b$$

$$\text{where } |V_{pfL, pu}| = \text{Magnitude of prefault voltage at the fault point in p.u.}$$

$$|I_{fL, pu}| = \text{Magnitude of short circuit interrupting current at the fault in p.u.}$$

**Here the short circuit interrupting MVA is a three phase power rating.**

Numerical.

A generator connected through a five cycle circuit breaker to a transformer, is rated at 100 MVA, 18 kV with reactances  $X_d'' = 20\%$ ,  $X_d' = 25\%$  and  $X_d = 110\%$ . It is operated on no-load and at rated voltage. When a 3 -phase fault occurs between the breaker and the transformer, find

- A. Short circuit current in circuit breaker
- B. The initial symmetrical rms current in the circuit breaker.
- C. The maximum possible dc component of the short circuit current in the breaker.
- D. The current to be interrupted by the breaker
- E. The Interrupting MVA

Let us choose the generator ratings as base values.

Therefore  $MVA_b = 100MVA$ ,  $kV_b = 18kV$

$$\text{Base Current} = I_b = \frac{kVA_b}{\sqrt{3}kV_b}$$

$$= (100 \times 1000) / \sqrt{3} \times 18$$

$$= 3207.5A$$

### A) To find short circuit currents in circuit breaker

The circuit breaker clears the faults in 5 cycles and so we can assume that the circuit opens during transient period of the fault.

Therefore the fault currents that may flow through circuit breaker are maximum momentary short circuit current, subtransient fault current and transient fault current.

If the fault exists even after 5 cycles (i.e., if the fault is not cleared by opening the circuit in 5 cycles) then steady state fault current will also flow through the circuit breaker.

The generator is running on no-load and so the internal emfs during subtransient and transient state are same as steady state induced emf. The generator is operated at rated voltage and so

$$E_g = E_g' = E_g'' = 1 \text{ p.u.}$$

Given that

$$X_d'' = 20\% = 0.2 \text{ p.u.}$$

$$X_d' = 25\% = 0.25 \text{ p.u.}$$

$$X_d = 110\% = 1.1 \text{ p.u.}$$

Subtransient fault current in generator excluding DC-offset current  $I_g'' = \frac{E_g''}{jX_d''}$

$$I_g'' = \frac{E_g''}{jX_d''} = \frac{1 \angle 0^\circ}{j0.2}$$

In polar form,  $j = 1 \angle 90^\circ$ .

$$I_g'' = \frac{E_g''}{jX_d''} = \frac{1 \angle 0^\circ}{j0.2} = \frac{1 \angle 0^\circ}{0.2 \angle 90^\circ} = 5 \angle -90^\circ \text{ p.u.}$$

Transient fault current in generator excluding DC-offset current

$$I_g' = \frac{E_g'}{jX_d'} = \frac{1\angle 0^\circ}{j0.25}$$

$$= \frac{1\angle 0^\circ}{0.25\angle 90^\circ}$$

$$= 4\angle -90^\circ \text{ p.u.}$$

Steady State fault current in generator excluding DC-offset current

$$I_g = \frac{E_g}{jX_d} = \frac{1\angle 0^\circ}{j1.1}$$

$$= \frac{1\angle 0^\circ}{1.1\angle 90^\circ} = 0.9091\angle -90^\circ \text{ p.u.}$$



The maximum momentary short circuit current is obtained by multiplying  $|I_g''|$  by 1.6, to account for dc-offset current

$$\text{Maximum momentary short circuit current} = 1.6 * |I_g''| = 1.6 \times 5 = 8 \text{ p.u.}$$

The maximum momentary short circuit current is obtained by multiplying  $|I_g''|$  by 1.6, to account for dc-offset current

$$\text{Maximum momentary short circuit current} = 1.6 * |I_g''| = 1.6 \times 5 = 8 \text{ p.u.}$$

The actual values of fault currents can be obtained by multiplying the p.u. current by base current

$$\text{Actual value of subtransient fault current} = |I_g''| * I_b = 5 \times 3207.5 = 16037.5 = 16.0375 \text{ kA}$$

$$\text{Actual value of transient fault current} = |I_g'| * I_b = 4 \times 3207.5 = 12830 \text{ A} = 12.83 \text{ kA}$$

$$\text{Actual value of Steady State fault current} = |I_g| * I_b = 0.9091 \times 3207.5 = 2915.9 \text{ A} = 2.9159 \text{ kA}$$

$$\text{Actual value of Maximum momentary short circuit current} = 8 * I_b = 8 \times 3207.5 = 25660 \text{ A} = 25.66 \text{ kA}$$

## B) To find initial symmetrical rms current.

The initial symmetrical rms current is the subtransient fault current excluding DC offset current,  
i.e., it is same as  $I_g''$ .

Initial symmetrical rms current =  $5 \angle -90^\circ$  p.u. or 16.0375kA

### C) To find maximum possible dc component

The maximum possible dc component of the short circuit current in the breaker is the difference between maximum momentary current and initial symmetrical short circuit current.

$$\begin{aligned}\text{Maximum possible dc component of the short circuit current} &= 25.66 - 16.0375 \\ &= 9.6225 \text{ kA}\end{aligned}$$

#### d) To find the interrupting current

The interrupting current of the circuit breaker can be estimated from the transient current by multiplying it by a suitable constant to account for dc-offset current at the time of interruption. For 5 cycles circuit breaker the multiplying factor is 1.1

$$\begin{aligned}\text{p.u value of short circuit interrupting current} &= | \text{p.u. value of transient fault current} | \times 1.1 \\ &= 4 \times 1.1 = 4.4\end{aligned}$$

$$\begin{aligned}\text{Actual value short circuit interrupting current} &= \text{p.u. value} \times \text{base current} \\ &= 4.4 \times 3207.5 \\ &= 14113 \text{ A} = 14.113 \text{ kA.}\end{aligned}$$

### e) To find interrupting MVA

The short circuit interrupting MVA = (p.u. value of Prefault Voltage) x (p.u. value of interrupting current) x Base MVA

$$= 1.0 \times 4.4 \times 100$$

$$= 440 \text{ MVA}$$



A 25MVA, 13.8kV generator with  $X_d''=15\%$  is connected through a transformer to a bus that supplies four identical motors as shown in fig. 2.18. Each motor has  $X_d''=20\%$  &  $X_d'=30\%$  on a base of 5MVA, 6.9kV. The three phase rating of the transformer is 25MVA, 13.8-6.9 kV. With a leakage reactance of 10%. The bus voltage at the motors is 6.9kV when a three-phase fault occurs at the point P. For the fault specified determine:

- The subtransient current in the fault.
- The subtransient current in the breaker A.
- The momentary current in breaker A.
- The current to be interrupted by breaker A in 5 cycles.

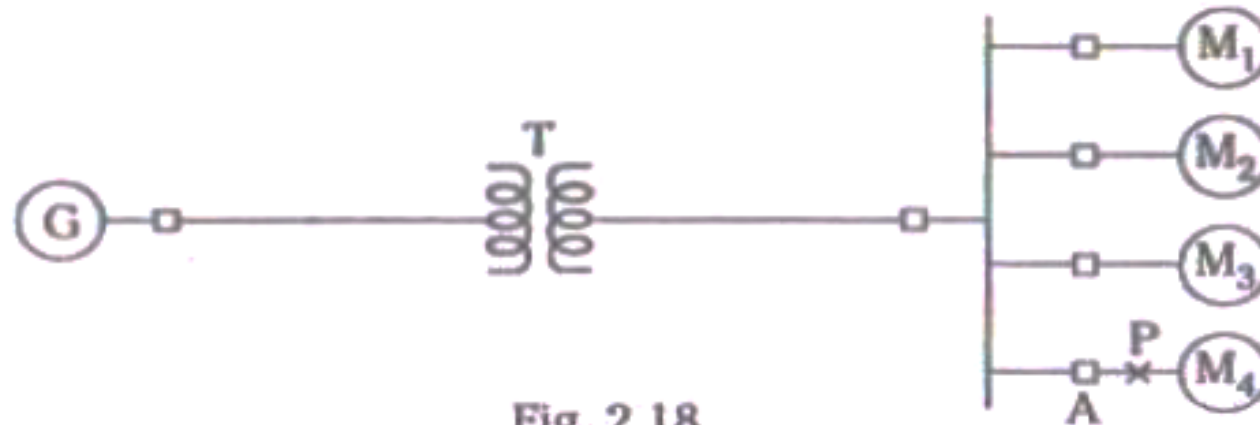


Fig. 2.18

Solution: base values: base MVA=25, base kV in the generator circuit=13.8  
 base voltage on the motor side=13.8×6.9/13.8=6.9

Reactance of generator G:

$X_{dG}'' = j0.15$  (same as old p.u value in given because base values have been chosen on the same machine ratings)

$X_{dG}' = j0.15$  (same as subtransient reactance as it is not specified in data).

Reactance of transformer T:

$X_T = j0.1$  (same as old p.u value in given because base values have been chosen on the same machine ratings)

Reactances of motors:

$$\begin{aligned} X_{dM,p.u,new}'' &= X_{dM,p.u,old}'' \times \left( \frac{(MVA)_{B, new}}{(MVA)_{B, old}} \right) \times \left( \frac{(kV)_{B, old}^2}{(kV)_{B, new}^2} \right) \\ &= j0.2 \times (25 / 5) \times (6.9^2 / 6.9^2) \\ &= j 1.0 \text{ p.u} \end{aligned}$$

$$\begin{aligned} X_{dM,p.u,new}' &= X_{dM,p.u,old}' \times \left( \frac{(MVA)_{B, new}}{(MVA)_{B, old}} \right) \times \left( \frac{(kV)_{B, old}^2}{(kV)_{B, new}^2} \right) \\ &= j0.3 \times (25 / 5) \times (6.9^2 / 6.9^2) \\ &= j 1.5 \text{ p.u} \end{aligned}$$

The prefault voltage at the point P is  $6.9\text{kV} = 6.9/6.9 = 1\text{p.u}$  and the base current in the  $6.9\text{kV}$  circuit is

$$I_b = (1000 \times \text{base power}) / (\sqrt{3} \times \text{base voltage}) = (1000 \times 25) / (\sqrt{3} \times 6.9) = 2091.8\text{A}$$

The reactance diagram with subtransient values of the reactance marked is shown in fig 2.19.

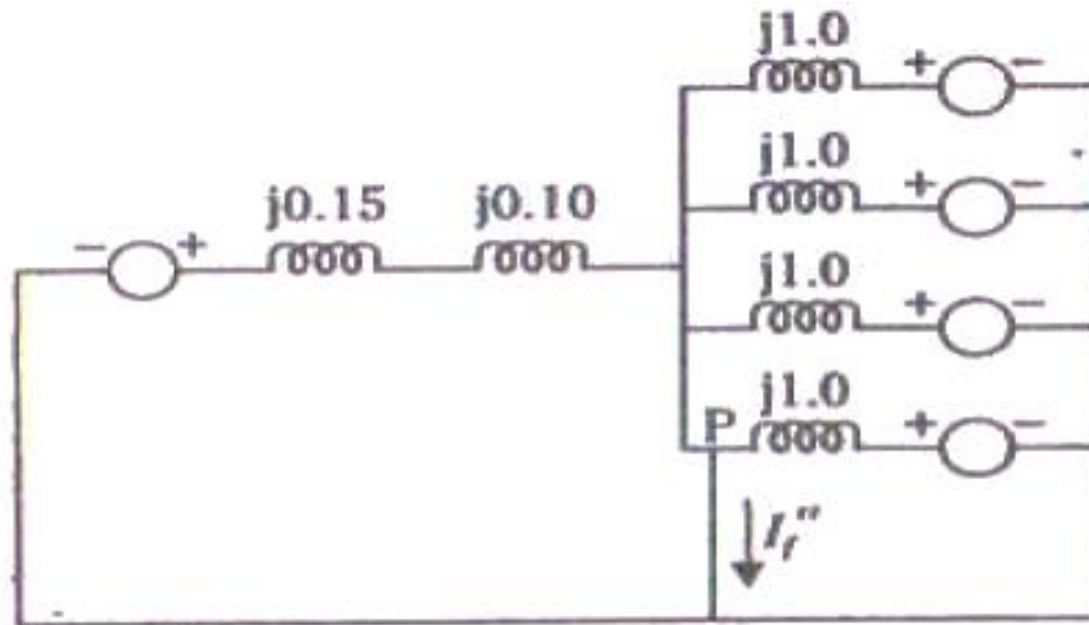


Fig. 2.19

a) therefore sub-transient fault current,  $I_f'' = 4 \times (1/j1.0) + (1/j0.25) = -j8 \text{ p.u}$

The absolute value of the current is  $I_f'' = -j8 \times 2091.8 = -j16734.4 \text{ A}$

b) therefore sub-transient current in breaker A,  $I'' = 3 \times (1/j1.0) + (1/j0.25) = -j7 \text{ p.u}$

The absolute value of the current is  $I'' = -j7 \times 2091.8 = -j14642.6 \text{ A}$

c) To find the momentary current in the breaker A, we must account for the d.c. Offset current. This is done empirically as follows:

Momentary current through breaker A =  $1.6 \times 14642.6 = 23428.16 \text{ A}$

d) To compute the current to be interrupted by the breaker A, it is required to obtain the transient reactance model of the system. This is shown in fig

The current to be interrupted by the breaker A now is  $= 3 \times (1/j1.5) + (1/j0.25) = -j6 \text{ p.u}$

Allowance is made for the d.c. Offset current by multiplying with a factor 1.1 (see table 2.1). therefore the current to be interrupted is,

$1.1 \times 6 \times 2091 = 13805.88 \text{ A}$

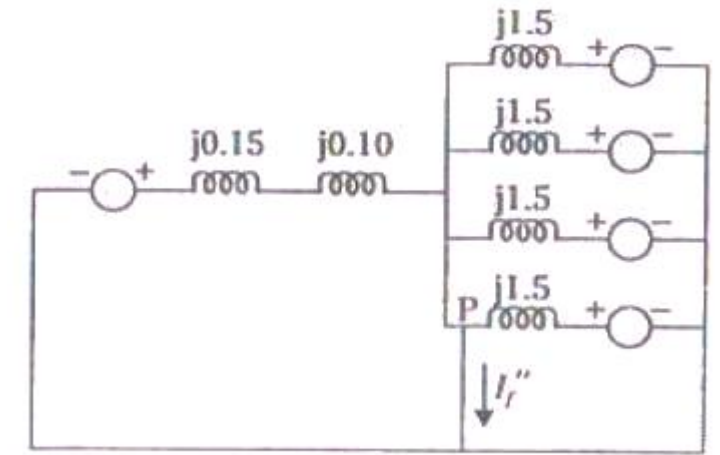


Fig. 2.20

A three phase, 5MVA, 6.6kV alternator with reactance of 8% is connected to a feeder of series impedance of  $(0.12+j0.48)$  ohms/phase per km. The transformer is rated at 3MVA, 6.6kV/33kV and has a series reactance of 5%. Determine the fault current supplied by the generator operating under no-load with a voltage of 6.9kV, when a three phase symmetrical fault occurs at a point 15km along the feeder.

Solution:

The single line diagram of the power system. let F be the point of occurrence of the fault.

Base values: Let us chose the generator rating as base values.

base MVA= 5, base kV on the generator=6.6

base kV on the transmission line= $6.6 \times 33 / 6.6 = 33$

Reactance of generator:

$X_G = 8\% = j0.08$  p.u

Reactance of transformer T: (calculated secondary side of it)HV or HT

$$X_{T, \text{ new }} = X_{T, \text{ old }} \times \left( \frac{(MVA)_{B, \text{ new }}}{(MVA)_{B, \text{ old }}} \right) \times \left( \frac{(kV)_{B, \text{ old }}^2}{(kV)_{B, \text{ new }}^2} \right)$$

$$= j0.05 \times (5 / 3) \times (33^2 / 33^2)$$

$$= j 0.833 \text{ p.u}$$

Impedance of the feeder:

$$\begin{aligned}
 Z_{TL,p.u} &= Z_{TL}(\Omega) \times (\text{MVA})_{B,new} / (\text{kV})^2_B \\
 &= (15 \times (0.12 + j0.48)) \times 5 / 33^2 \\
 &= 0.0083 + j0.033 \text{ p.u}
 \end{aligned}$$

To find  $E_G$  and  $V_{pf}$ :

Actual value of induced emf,  $E_G = 6.9 \text{ kV}$

p.u value of induced emf,  $E_G = \text{actual value} / \text{base value}$   
 $= 6.9 / 6.9 = 1.0455 \text{ p.u}$

The prefault voltage  $V_{pf}$  at fault point F is the voltage under no-load =  $34.5 \text{ kV}$   
 therefore,

prefault voltage  $V_{pf} = 34.5 \text{ kV}$

The p.u value of prefault voltage,  $V_{pf} = \text{actual value} / \text{base value} = 34.5 / 33 = 1.0455 \text{ p.u}$

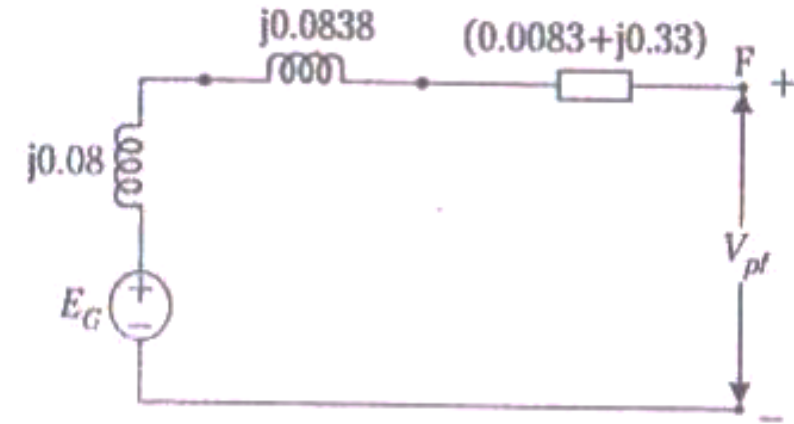


Fig. 2.30



To find fault current:

The Thevenin's equivalent circuit of the system in fig 2.30 as seen from the fault point F is shown in fig 2.31. Here

$$V_{TH} = 1.045 \angle 0^\circ$$

$$Z_{TH} = j0.08 + j0.0833 + (0.0083 + j0.33) = 0.0083 + j0.4933$$

The fault in the feeder can be represented by a short circuit as shown in fig. 2.32.

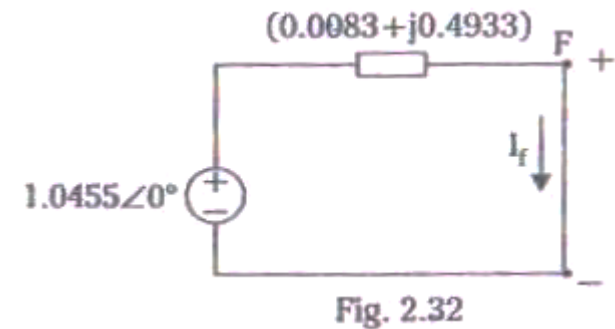
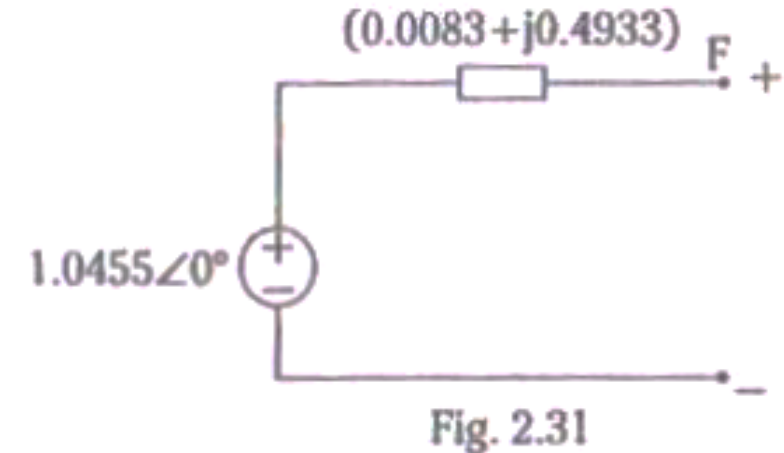
Now the current  $I_f$  through the short circuit is the fault current.

Therefore,

p.u value of fault current,

$$I_f = V_{TH} / Z_{TH} = (1.045 \angle 0^\circ) / (0.0083 + j0.4933) = \underline{2.12 \angle -89^\circ}$$

p.u



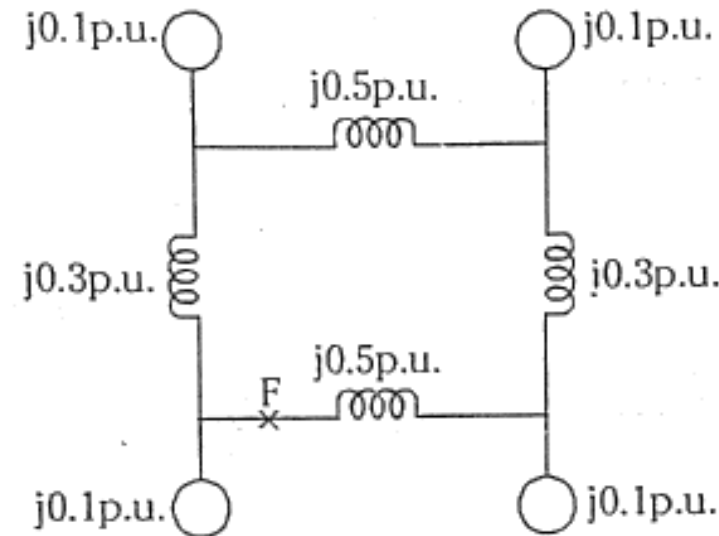
therefore,

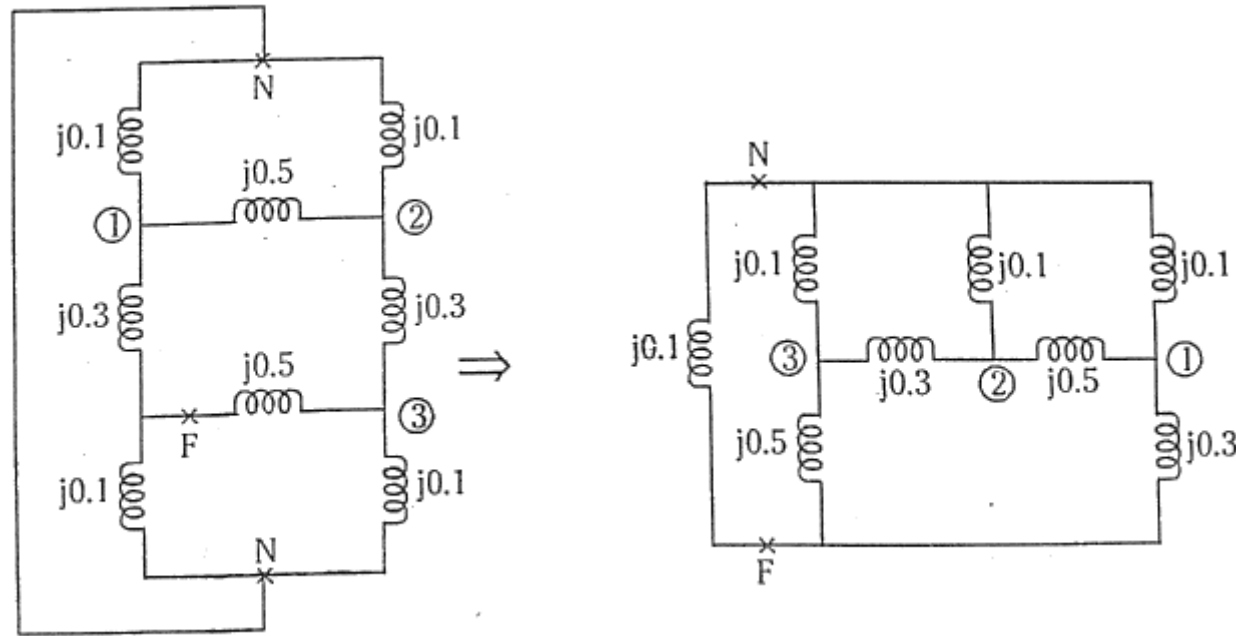
actual value of fault current,  $I_f = \text{p.u value} \times \text{base current}$

$\text{base current} = (1000 \times \text{base power}) / (\sqrt{3} \times \text{base voltage}) = (1000 \times 5) / (\sqrt{3} \times 33) = 87.47 \text{ A}$

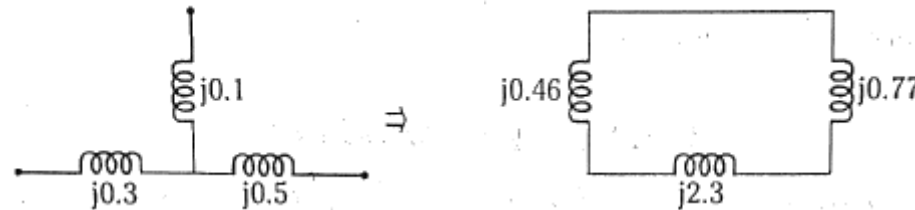
$I_f = (2.12 \angle -89^\circ) \times 87.47 = 185.45 \angle -89^\circ \text{ A}$

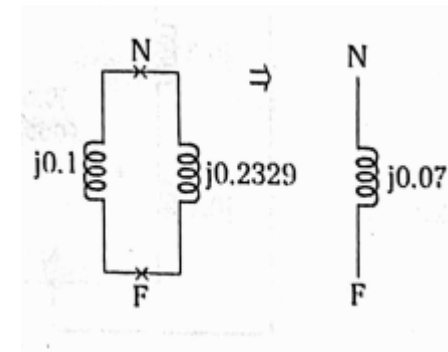
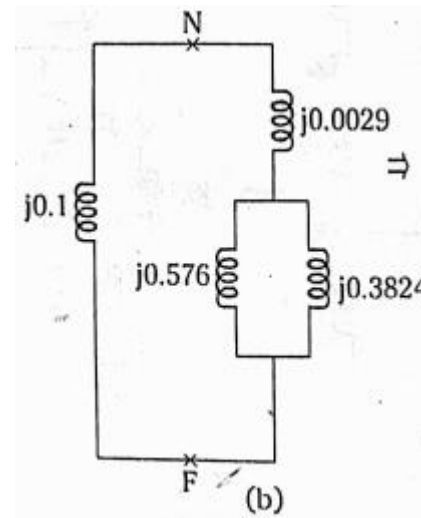
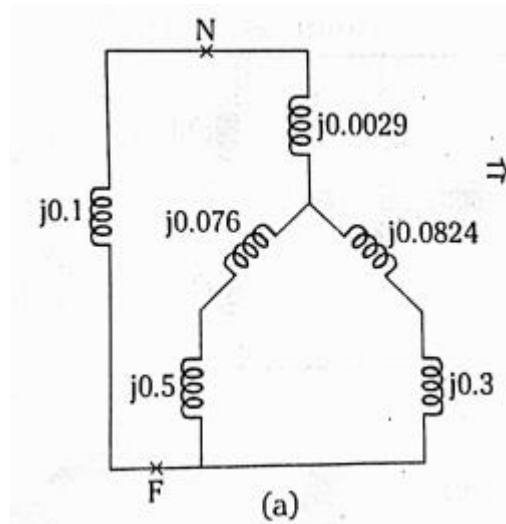
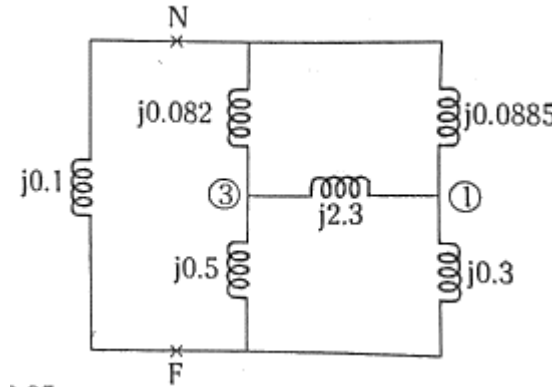
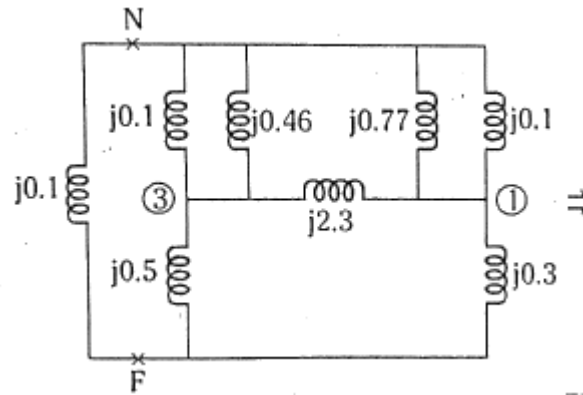
**Example 2.7 : Determine the fault MVA, if a fault takes place at F in the diagram shown (Fig. 2.32). The p.u. values of reactance are given with 100 MVA as base.**





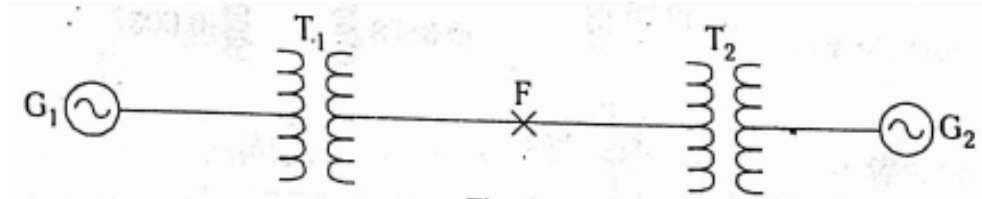
$$|MVA|_{S.C.} = \frac{|MVA|_B}{|Z_{p.u.}|}$$





$$|MVA|_{s.c.} = \frac{100}{0.07} = 1428.57 \text{ MVA}$$

Generator G1 and G2 are identical and rated 11 kV, 20 MVA and have a transient reactance of 0.25 p.u. at own MVA base. The transformer T1 and T2 are also identical and are rated 11 - 66 kV, 5 MVA and have a reactance of 0.06 p.u. to their own MVA base. The tie-line is 50 km long : each conductor has a reactance of 0.848 ohm /km. The three - phase fault is assumed at F, 20 km from generator G1 as shown in Fig. Find the short circuit current.



### Base values

base MVA be 20 MVA. Base voltage on the generator sides = 11 kV. Base voltage on the line =  $11 \times \frac{66}{11} = 66$  kV.

### Reactance of generators

$$X_{G1} = X_{G2} = j0.25 \text{ p.u.}$$

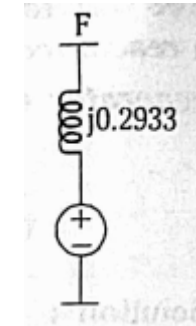
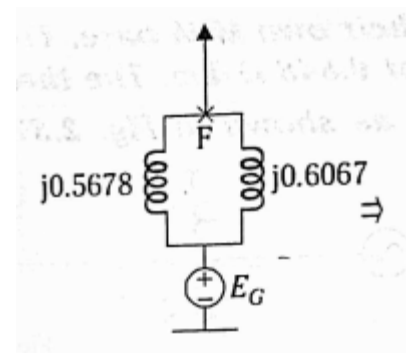
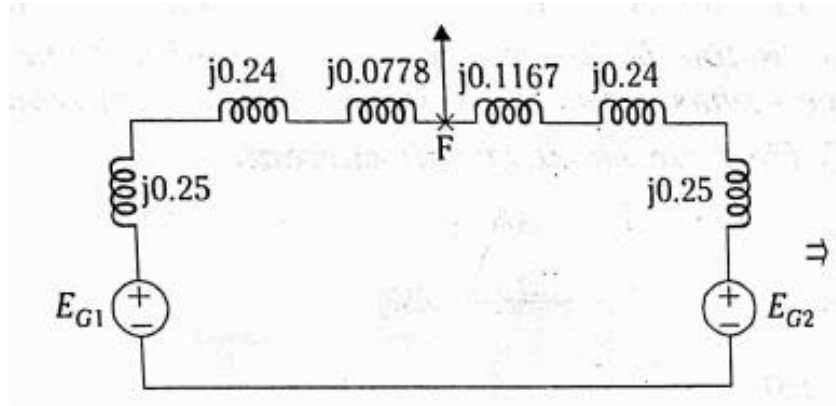
### Reactance of transformers

$$X_{T1, new} = X_{T2, new} = X_{old} \times \frac{(MVA)_{B, new}}{(MVA)_{B, old}} \times \frac{(kV)_{B, old}^2}{(kV)_{B, new}^2} = j0.06 \times \frac{(20)}{(5)} \times \frac{(66)^2}{(66)^2} = j0.24 \text{ p.u.}$$

### Reactance of transmission line

$$\text{20 km long : } X_{20} = (j0.848 \times 20) \times \frac{(MVA)_B}{(kV)_B^2} = (j0.848 \times 20) \times \frac{(20)}{(66)^2} = j0.778 \text{ p.u.}$$

$$30 \text{ km long} : X_{30} = (j0.848 \times 30) \times \frac{20}{(66)^2} = j0.1167 \text{ p.u.}$$



$$|MVA|_{S.C.} = \frac{(MVA)_B}{|Z_{p.u.}|} = \frac{20}{0.2933} = 68.18 \text{ MVA.}$$

$$|MVA|_{S.C.} = \sqrt{3} \times |V_B| \times |I_f| \times 10^{-3}$$

$$\text{where } V_B = 66 \text{ kV.}$$

$$\text{Thus } |I_f| = \frac{|MVA|_{S.C.}}{\sqrt{3} \times |V_B| \times 10^{-3}} = \frac{68.18}{\sqrt{3} \times 66 \times 10^{-3}} = 596.4 \text{ A.}$$